U.S.N

P.E.S. College of Engineering, Mandya - 571 401

(An Autonomous Institution affiliated to VTU, Belagavi)

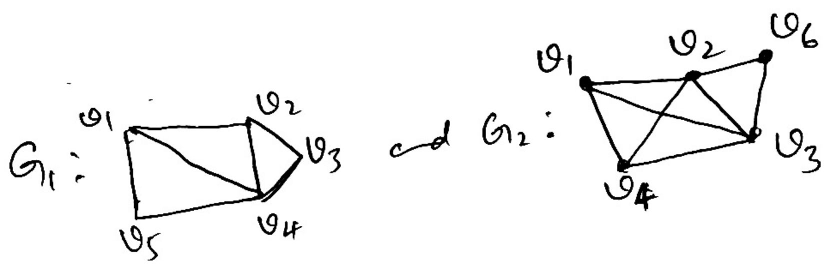
Third Semester, B.E. - Computer Science and Engineering

Semester End Examination; Dec. - 2019

Discrete Mathematical Structures

Time: 3 hrs *Max. Marks: 100*

Note: i) PART - A is compulsory. **Two** marks for each question.
ii) PART - B: Answer any **Two** sub questions (from a, b, c) for Maximum of **18 marks** from each unit.

Q. No.	Questions	Marks
	I : PART - A	**10**
I a.	For any two propositions p, q. Prove that $(p \rightarrow q) \Leftrightarrow (\neg p) \vee q$.	2
b.	Obtain the recursive definition for the sequence (a_n) given by $a_n = n\,(n + 2)$.	2
c.	Let $f : A \rightarrow B$ be a function and C and D be arbitrary non-empty subsets of B. Then prove that $f^{-1}[C \cap D] = f^{-1}(C) \cap f^{-1}(D)$	2
d.	In a class of 52 students, 30 are studying C++, 28 are studying PASCAL and 13 are studying both languages. How many in this class are studying at least one of these languages? How many are studying neither of these languages?	2
e.	Given :  Find; $G_1 \cup G_2$.	2
	II : PART - B	**90**
	UNIT - I	**18**
1 a.	Prove that for any three propositions p, q, r $[(p \vee q) \rightarrow r] \Leftrightarrow [(p \rightarrow r) \wedge (q \rightarrow r)]$.	9
b.	Test the validity of the following argument; $p \rightarrow (q \rightarrow r)$ $\neg q \rightarrow \neg p$ p ―――――― $\therefore r$	9
c.	Test the validity of the following argument If I study. I will not fail in the examination If I do not watch TV in the evenings. I will study. I failed in the examination. ―――――― $\therefore$ *I must have watched TV in the evenings*	9

UNIT - II **18**

2 a. Prove by mathematical induction 9

$$1^2 + 2^2 + 3^2 + ------ + n^2 = \frac{1}{6} n(n+1)(2n+1)$$

b. i) A sequence (a_n) is defined recursively by $a_1 = 4,\ a_n = a_{n-1} + n$ for $n \geq 2$. Find a_n in explicit form.

ii) Find the number of ways that a judge can award first, second and third places in a contest with eighteen contestants. 9

c. Find the number of ways that three American, four Frenchmen, four Danes and two Italians can be seated in a row so that those of the same nationality sit together 9

UNIT - III **18**

3 a. State pigeon hole principle and extended pigeon hole principle. Find the least number of ways of choosing three different numbers from 1 to 10 so that all choices have the same sum. 9

b. Draw the Hasse diagram representing the positive divisors of 36, 50. 9

c. Let "R" be a relation defined on A = {1, 2, 3, 4, 5} as $(a, b) \in$ R iff $a \equiv b \pmod 3$

i) Prove that R is an equivalence relation

ii) Write the relation matrix M(R) 9

iii) Draw the Digraph

iv) Find the partition induced by R on A

UNIT - IV **18**

4 a. Find the rook polynomial of the board that is obtained from a 3 x 3 board by deleting the middle square in the first row and the first and the last squares in the third row. 9

b. i) Solve $a_n = 4\, a_{n-1}\ n \geq 1$ given $a_0 = 3$

ii) Solve the recurrence relation $a_n = 3\, a_{n-1} - 2\, a_{n-2}$ for $n \geq 2\ \ a_0 = 5,\ a_1 = 3.$ 9

c. Determine the number of integers between 1 to 300 which are,

i) Divisible by two of 5, 6, 8 9

ii) Divisible by at least two of 5, 6, 8

UNIT - V **18**

5 a. Define planar graph and prove that "A connected planar with n vertices and e edges has exactly $e - n + 2$ regions". 9

b. Define the following terms :

i) Euler graph

ii) Hamiltonian graph 9

iii) Bipartite graph

c. Define prefix code. Construct an optimal prefix code tree and code for the message "HAPPY INDEPENDENCE DAY". 9

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