U.S.N | | | | | | | | | | |

P.E.S. College of Engineering, Mandya - 571 401

(An Autonomous Institution affiliated to VTU, Belagavi)

Third Semester, B.E. - Computer Science and Engineering
Semester End Examination; March - 2021
Discrete Mathematical Structures

Time: 3 hrs *Max. Marks: 100*

Note: *Answer **FIVE** full questions, selecting **ONE** full question from each unit.*

UNIT - I

1 a. Find the number of license plates created which contains two English alphabets followed by four digits, i) With repetition and ii) Without repetition. 6

b. How many arrangements are these of all the letters in MASSASAUGA such that four A's are together? 6

c. i) Determine the coefficient of $x^2 y z^2$ in the expansion $(2x+y+3z)^5$

ii) Determine the coefficient of $x^2 y^2 z^3$ in the expansion of $(3x-2y-4z)^7$ 8

2 a. Use the laws of set theory, simplify the expression $\overline{\overline{\overline{(A \cup B)} \cap C} \cup \overline{B}}$. 3

b. Over the internet data are transmitted in structured blocks of bits called datagram's?

i) In how many ways can the letters in DATAGRAM be arranged? 4

ii) For the arrangements of part (i) how many have all three 'A' together?

c. The board of directors of pharmaceutical corporations has 10 members. An upcoming stock holders meeting is scheduled to approve a new state of company officers (chosen from the 10 board members):

I) How many different slates consisting of a president, vice-president, secretary and treasurer can be the board present to the stock holders for their approval? 6

II) Three members of the board of directors are physicians. How many slates from part (I) have; i) A physician nominated for presidency

ii) Exactly one physician appearing on the state

d. A manufacturer of 2000 automobile batteries is carried about defective terminals and defective plates. If 1920 of batteries have neither defect, 60 have defective plates and 20 have both defects. How many batteries have defective terminals? 7

UNIT - II

3 a. Prove the following logical equivalence:

$(p \to q) \wedge [\neg q \wedge (r \vee \neg q)] \equiv (q \vee p)$ 6

b. Find whether the following argument is valid: 8

If a triangle has two equal sides, then it is isosceles,

If a triangle is isosceles, thus it has two equal angle,

<u>The triangle ABC does not have two equal angles.</u>

$\therefore$ ABC does not have two equal sides.

c. Convert the following statement to symbolic form and also write its negation: "For all x, if x is odd, then x^2-1 is even". 6

4 a. Using the laws of logic, simplify the given statement $(p \to q) \wedge [\neg q \wedge (r \vee \neg q)]$. 6

b. Establish the validity of the following argument: 8

$p \wedge q$
$p \to (r \wedge q)$
$r \to (s \vee t)$
$\neg s$

$\therefore t$

c. Prove that the following argument is valid where in c is specified element of the universe. 6

$\forall x[p(x) \to q(x)]$
$\forall x[q(x) \to r(x)]$
$\neg r(c)$

$\therefore \neg p(c)$

UNIT - III

5 a. Prove by mathematical induction that $1^2 + 3^2 + 5^2 + ----+(2n-1)^2 = \frac{1}{3}n(2n-1)(2n+1)$. 6

b. A sequence $\{c_n\}$ is defined recursively by $c_n = 3c_{n-1} - 2c_{n-2}$ for all $n \geq 3$ with $C_1 = 5$ and $C_2 = 3$ as initial conditions. Show that $c_n = -2^n + 7$. 6

c. If $A = \{1, 2, 3, 4\}$, $B = \{2, 5\}$, $C = \{3, 4, 7\}$
Determine AXB, BXA, $A \cup (BXC)$, $(A \cup B)XC$, $(AXC) \cup (BXC)$. 8

6 a. i) For $n \geq 2$ any sets $A_1, A_2, -----A_n \in U$
Prove that $\overline{A_1 \cup A_2 ----\cup A_n} = \overline{A_1} \cap \overline{A_2} \cap --- \cap \overline{A_n}$. 7

ii) The Fibonacci numbers are defined recursively by $F_0 = 0$, $F_1 = 1$ and $F_n = F_{n-1}+F_{n-2}$ for $n \geq 2$. Evaluate F_2, F_5, and F_7.

b. Define permutation function, hashing function and characteristic function. 7

c. i) Let f, g: $R \to R$ where $g(x) = 1-x+x^2$ and $f(x) = ax+b$. If $gof(x) = 9x^2-9x+3$, determine a, b. 6

ii) Given $p = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 4 & 3 & 1 & 5 & 6 \end{bmatrix}$. Compute p^{-1}, p^2.

UNIT - IV

7 a. Let $A = \{1, 2, 3, 4\}$ and $R = \{(1, 1), (1, 2), (2, 1), (3, 1), (3, 3), (1, 3), (4, 1), (4, 4)\}$ be a relation on A. Is R is an equivalence relation? 6

b. Draw Hasse diagram for divisors of 36. 8

c. Let $A = \{1, 2, 3, 4, 6\}$ and R be a relation defined by aRb if and only if a is multiple of b. 6
Represent the relation R as matrix and draw its digraph.

8 a. Consider the following relation on set $A = (1, 2, 3\}$, $R_1 = \{(1, 1), (1, 2), (1, 3), (3, 3)\}$ and $R_2 = \{(1, 1), (1, 2), (2, 1), (2, 2), (3, 3)\}$. Which of these are; 6

i) Reflexive ii) Symmetric iii) Transitive iv) Anti symmetric

b. The digraph for a relation on the set $A = \{1, 2, 3, 4\}$ is shown in Fig. 8(b).

i) Verify that (A, R) is a Poset and find its Hasse diagram ii) Topological sort (A, R) 6

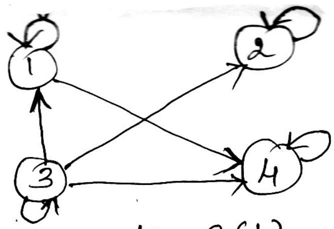

fig 8(b).

c. For the posets shown in the following Fig.8(c) find: 8

i) All upper bounds ii) L∪B and GLB of the set $B = \{3, 4, 5\}$

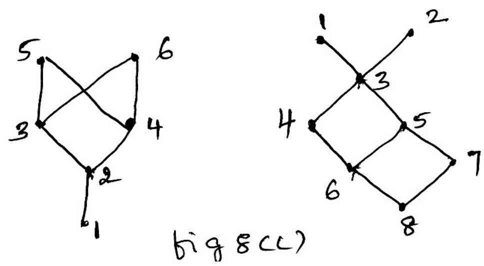

fig 8(c)

UNIT - V

9 a. Define cyclic group and show that $(G, *)$ whose multiplication table is given is cyclic. 6

*	a	b	c	d	e	f
a	a	b	c	d	e	f
b	b	c	d	e	f	a
c	c	d	e	f	a	b
d	d	e	f	a	b	c
e	e	f	a	b	c	d
f	f	a	b	c	d	e

b. Explain Lagragis theorem. If G is a group of order n and $a \in$ G, prove that $a^n = e$. 6

c. Define the following terms with respect to coding theory: 8

i) Parity checkcode ii) Hamming distance iii) Group code iv) Generator matrix

10 a. The parity-check matrix for an encoding function $Z_2^3 \rightarrow Z_2^6$ is given by 6

$$H = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

i) Determine associated gennerator matrix

ii) Does this code correct all single error in transmission

b. Let G is a group and $a, b \in$ G, prove that 6

$i)\ (a^{-1})^{-1} = a$ $ii)\ (ab)^{-1} = b^{-1}\,a^{-1}$

c. Define Sub group. If H, K are sub group of G, prove that $H \wedge K$ is also subgroup. 8

* * *