

**P.E.S. College of Engineering, Mandya - 571 401***(An Autonomous Institution affiliated to VTU, Belagavi)***First Semester, B.E. - Semester End Examination; Dec. - 2019****Engineering Mathematics - I***(Common to all Branches)*

Time: 3 hrs

Max. Marks: 100

Course Outcome's*The Students will be able to:*

CO1 - Apply the knowledge of calculus to solve problems related to polar curves and its applications in determining the bentness of a curve.

CO2 - Explain mean value theorems and evaluate the indeterminate form and power series using Taylors and Maclaurin's series.

CO3 - Differentiate the function of several variables differentiate the composite function. Evaluate the vector differentiation.

CO4 - Evaluate some standard integrals by applying reduction formula and solve application problems. Solve differential equations of first order and solve application problems in engineering field.

Note: i) Part - A is compulsory, one question from each unit

ii) Part - B : Answer Two sub-questions for Maximum of 18 marks from each unit.

Q. No.	Questions	Marks	BLs	COs
I : PART - A		10		
I. a.	Find the n^{th} derivative of $\sin 4x \cos 2x$	2	L1	CO1
b.	State the Rolle's theorem.	2	L1	CO1
c.	Find the total differential of the function $u = x^3 + xy^2 + x^2y + y^3$.	2	L1	CO3
d.	Evaluate $\int_0^{\pi/2} \sin^6 x \cdot \cos^5 x dx$	2	L1	CO4
e.	Show that the differential equation $(y^3 - 3x^2y)dx - (x^3 - 3xy^2)dy = 0$ is exact.	2	L1	CO5
II : PART - B		90		
UNIT - I		18		
1 a.	State Leibnitz theorem. If $y = (\sin^{-1} x)^2$ Show that, $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2y_n = 0$	9	L1	CO1
b. i)	Find the angle between the radius vector and the tangent for the polar curve $r = a(1 - \cos \theta)$	9	L1	CO1
ii)	Find the pedal equation of the curve $r(1 - \cos \theta) = 2a$			
c.	Define radius of curvature. Find the radius of curvature for the curve. $y^2 = \frac{4a^2(2a-x)}{x}$ Where the curve meets the x -axis.	9	L2	CO1
UNIT - II		18		
2 a. i)	State the Lagrange's mean value theorem			
ii)	Verify the Cauchy's mean value theorem for the function $\sqrt{x+9}$ and \sqrt{x} in $[0, 16]$	9	L2	CO2
b. i)	State the Taylor's series expansion about the point $x=a$ upto the fourth degree term	9	L2	CO2
ii)	Expand $\log(\sec x)$ up to the term containing x^6 using Maclaurin's series.			

c. Evaluate the following limits:

i) $\lim_{x \rightarrow a} \frac{x^x - a^x}{x^a - a^a}$

ii) $\lim_{x \rightarrow \frac{\pi}{2}} (\sin x)^{\tan x}$

9 L2 CO2

UNIT - III

18

3 a. If $u = \tan^{-1}\left(\frac{x^3 + y^3}{x - y}\right)$ then prove that using,

Euler's theorem

9 L2 CO3

i) $xu_x + yu_y = \sin 2u$

ii) $x^2u_{xx} + 2xyu_{xy} + y^2u_{yy} = \sin 4u - \sin 2u$

b. i) Define velocity and acceleration and also give the examples.

ii) Find the directional derivatives of,

9 L2 CO3

$\phi = x^2yz + 4xz^2$ at $(1, -2, -1)$ along $2i - j - 2k$.

c. i) Define solenoidal and irrotational vector fields.

ii) Find the div \vec{F} and curl \vec{F} where $\vec{F} = \nabla(x^3 + y^3 + z^3 - 3xyz)$

9 L2 CO3

UNIT - IV

18

4 a. Obtain the reduction formula for $\int \sin^n x dx$ and $\int_0^{\pi/2} \sin^n x dx$ where n is a positive

9 L2 CO4

integer and find $\int_0^{\pi/2} \cos^6 x dx$

b. i) Trace the curve $y^2(a - x) = x^3, a > 0$ (cissoid).

9 L2 CO4

ii) Write the shape of the cardioid $r = a(1 + \cos \theta)$

c. Evaluate: $\int_0^1 \frac{x^\alpha - 1}{\log x} dx (\alpha \geq 0)$ using differentiation under the integral sign where α

9 L2 CO4

is the parameter and hence find $\int_0^1 \frac{x^3 - 1}{\log x} dx$

UNIT - V

18

5 a. i) Define Homogeneous differential equation.

ii) Solve: $\left(1 + e^{x/y}\right)dx + e^{x/y}\left(1 - \frac{x}{y}\right)dy = 0$

9 L2 CO5

b. i) State exact differential equation

ii) solve: $\frac{dy}{dx} + 3x^2y = x^5e^{x^3}$

9 L2 CO5

c. If the air is maintained at 30°C and the temperature of the body cools from 80°C to 60°C in 12 minutes. Find the temperature of the body after 24 minutes.

9 L2 CO5