U.S.N

P.E.S. College of Engineering, Mandya - 571 401

(An Autonomous Institution affiliated to VTU, Belagavi)

Third Semester, B.E. - Semester End Examination; Dec. - 2019

Transform Calculus Fourier series and Numerical Technique

(Common to all Branches)

Time: 3 hrs *Max. Marks: 100*

Course Outcomes

The Students will be able to:

CO1: Apply the knowledge of calculus to solve problems related to polar curves and its applications in determining the bentness of a curve.

CO2: Explain mean value theorems and evaluate the indeterminate form and power series using Taylors and Maclaurin's series.

CO3: Differentiate the function of several variables differentiate the composite function. Evaluate the vector differentiation.

CO4: Evaluate some standard integrals by applying reduction formula and solve application problems. Solve differential equations of first order and solve application problems in engineering field.

Note: I) PART - A is compulsory, one question from each unit.

II) PART - B: Answer Two sub-questions for Maximum of 18 marks from each unit.

<table>
<tr><th>Q. No.</th><th>Questions</th><th>Marks</th><th>BLs</th><th>COs</th></tr>
<tr><td></td><td>I : PART - A</td><td>10</td><td></td><td></td></tr>
<tr><td>I a.</td><td>Construct the divided difference table for the data given below:
<table>
<tr><td>X</td><td>2</td><td>4</td><td>5</td><td>6</td></tr>
<tr><td>Y</td><td>10</td><td>96</td><td>196</td><td>350</td></tr>
</table></td><td>2</td><td>L1</td><td>CO1</td></tr>
<tr><td>b.</td><td>Write the first derivative of Newton's backward interpolation formula upto 4th degree terms.</td><td>2</td><td>L1</td><td>CO2</td></tr>
<tr><td>c.</td><td>Define complex form of Fourier Series of $f(x)$ having period 2π, where $-\pi < x < \pi$</td><td>2</td><td>L1</td><td>CO3</td></tr>
<tr><td>d.</td><td>Define Z-Transform of u_n.</td><td>2</td><td>L1</td><td>CO4</td></tr>
<tr><td>e.</td><td>Form the partial differential equation by eliminating the arbitrary constants
$2z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$</td><td>2</td><td>L1</td><td>CO4</td></tr>
<tr><td></td><td>II : PART – B</td><td>90</td><td></td><td></td></tr>
<tr><td></td><td>UNIT – I</td><td>18</td><td></td><td></td></tr>
<tr><td>1 a.</td><td>i) Define Extrapolation.
ii) From the following data estimate the number of students scoring the marks more than 40 but less than 45 marks.
<table>
<tr><td>Marks</td><td>30 - 40</td><td>40 - 50</td><td>50 - 60</td><td>60 - 70</td><td>70 - 80</td></tr>
<tr><td>No. of Students</td><td>31</td><td>42</td><td>51</td><td>35</td><td>31</td></tr>
</table></td><td>9</td><td>L1</td><td>CO1</td></tr>
<tr><td>b.</td><td>i) Write the Lagrange's inverse interpolation formula for $x = f(y)$.
ii) The following table gives the normal weights of babies during the first eight months of life.
<table>
<tr><td>Age (Months)</td><td>0</td><td>2</td><td>5</td><td>8</td></tr>
<tr><td>Weight (Pounds)</td><td>6</td><td>10</td><td>12</td><td>16</td></tr>
</table>
Contd…2</td><td>9</td><td>L2</td><td>CO1</td></tr>
</table>

<table>
<tr><td></td><td>Estimate the weight of the baby at the age of seven months using Lagrange's interpolation formula.</td><td></td><td></td><td></td></tr>
<tr><td>c.</td><td>i) Write Gauss's forward interpolation formula up to 4th degree terms.
ii) Apply Stirling's formula to find the cubic polynomial satisfying,
$f(-4) = -25, f(-2) = 1, f(0) = 3, f(2) = 29, f(4) = 127$ and hence find $f(3)$.</td><td>9</td><td>L3</td><td>CO1</td></tr>
<tr><td></td><td>UNIT – II</td><td>18</td><td></td><td></td></tr>
<tr><td>2 a.</td><td>Find maximum and minimum values of the function $y = f(x)$ from the following data using Newton's forward interpolation formula.
<table>
<tr><td>x</td><td>1</td><td>3</td><td>5</td><td>7</td><td>9</td></tr>
<tr><td>y</td><td>9</td><td>11</td><td>13</td><td>63</td><td>209</td></tr>
</table></td><td>9</td><td>L1</td><td>CO2</td></tr>
<tr><td>b.</td><td>i) Write the Simpson's $\left(\frac{1}{3}\right)^{rd}$ rule for $n = 6$.
ii) Use Simpson's $\left(\frac{3}{8}\right)^{th}$ rule to obtain the approximate value of,
$\int_0^{0.3} \sqrt{\left(1-8x^3\right)}dx$ by considering 6 equal intervals.</td><td>9</td><td>L3</td><td>CO2</td></tr>
<tr><td>c.</td><td>i) Write Boole's rule for $n = 8$.
ii) Evaluate $\int_0^1 \frac{x}{1+x^2}dx$ by Weddle's rule taking seven ordinates and hence find $\log_e 2$</td><td>9</td><td>L3</td><td>CO2</td></tr>
<tr><td></td><td>UNIT - III</td><td>18</td><td></td><td></td></tr>
<tr><td>3 a.</td><td>Obtain the Fourier Series for the function:
$f(x)=\begin{cases}-\pi & in \quad -\pi<x<0 \\ x & in \quad 0<x<\pi\end{cases}$. Hence deduce that $\frac{\pi^2}{8}=\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+.......$</td><td>9</td><td>L3</td><td>CO3</td></tr>
<tr><td>b.</td><td>i) Define cosine half range Fourier series $f(x)$ in $(0, \pi)$
ii) Obtain the sine half range Fourier series of $f(x)=\begin{cases}\frac{1}{4}-x \ in \ 0<x<\frac{1}{2} \\ x-\frac{3}{4} \ in \ \frac{1}{2}<x<1\end{cases}$</td><td>9</td><td>L3</td><td>CO3</td></tr>
<tr><td>c.</td><td>Express y as a Fourier series upto the second harmonics given the following data:
<table>
<tr><td>x</td><td>0</td><td>π/3</td><td>2π/3</td><td>π</td><td>4π/3</td><td>5π/3</td><td>2π</td></tr>
<tr><td>y</td><td>1.98</td><td>1.30</td><td>1.05</td><td>1.30</td><td>–0.88</td><td>–0.25</td><td>1.98</td></tr>
</table></td><td>9</td><td>L2</td><td>CO3</td></tr>
<tr><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td></td><td>Contd…3</td><td></td><td></td><td></td></tr>
</table>

	UNIT - IV	**18**		
4 a.	Find the Fourier Transform of, $f(x)=\begin{cases}1-x^2, \lvert x\rvert<1\\ 0, \quad \lvert x\rvert\ge 1\end{cases}$ Find the Fourier transform of *f(x)* and hence find the value of, $\int_0^1 \frac{x\cos x-\sin x}{x^3}dx$	9	L2	CO4
b.	Solve the integral equation, $\int_0^\infty f(\theta)\cos\alpha\theta\, d\theta=\begin{cases}1-\alpha, & 0\le\alpha\le 1\\ 0, & \alpha>1\end{cases}$ and Hence evaluate $\int_0^\infty \frac{\sin^2 t}{t^2}\, dt$	9	L3	CO4
c.	i) Find the Z-transform of $(n+1)^2$ ii) Solve by using Z-transforms : $Y_{n+2}-4y_n=0$ given that $y_0=0$ and $y_1=2$.	9	L2	CO4
	UNIT - V	**18**		
5 a.	i) Solve by direct integration $\frac{\partial^2 y}{\partial x\partial t}=0, \quad z=z(x,t)$ ii) Form the Partial differential equation by elimination the arbitrary function $\varphi(x+y+z, \quad x^2+y^2-z^2)=0$	9	L1	CO4
b.	i) Define Homogeneous particular equation. ii) Solve $x(y^2-z^2)p+y(z^2-x^2)q=z(x^2-y^2)$	9	L3	CO4
c.	Obtain the various possible solutions of the two dimensional Laplace equations $u_{xx}+u_{yy}=0$ by the method of separation of variables.	9	L3	CO4

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