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P.E.S. College of Engineering, Mandya - 571 401
(An Autonomous Institution affiliated to VTU, Belagavi)
Third Semester, B.E. - Information Science and Engineering
Semester End Examination; Dec. - 2019
Discrete Mathematics and Applications

Time: 3 hrs   *Max. Marks: 100*

Note: i) **PART - A** is compulsory. **Two** marks for each question.

ii) **PART - B**: Answer any **Two** sub questions (from a, b, c) for Maximum of **18 marks** from each unit.

Q. No.	Questions	Marks
	I : PART - A	**10**
I a.	Show that $(\sim p) \to (p \to q)$ is tautology.	2
b.	How many permutations of the letters in ABCDEFGH contain the string ABC?	2
c.	Let f:A→B be any function. Then prove that $I_{BO} f = f$.	2
d.	Suppose that in any colony of bacteria, the number of bacteria doubles every hour and after doubling two bacteria, die immediately. Develop a recurrence relation that gives number of bacteria alive after n hours.	2
e.	Draw a complete bipartite graph of $K_{2,3}$ and $K_{3,3}$	2
	II : PART - B	**90**
	UNIT - I	**18**

1 a. Show that the following premises are inconsistent:

i) If Nirmala misses many classes through illness then he fails high school

ii) If Nirmala fails high school, then he is un educated   9

iii) If Nirmala reads a lot of books then he is not uneducated.

iv) Nirmala misses many classes through illness and reads a lot of books

b. Identify the truth value of each statement, if the universe comprises all non-zero integers.

i) $\exists x \ \forall y \ [xy = 2]$ ii) $\forall x \ \exists y \ [xy = 2]$ 9

iii) $\exists x \ \exists y \left[(3x + y = 8) \wedge (2x - y = 7) \right]$ iv) $\exists x \ \exists y \left[(4x + 2y = 3) \wedge (x - y = 1) \right]$

c. It is true that the negation of a conditional statement is also a conditional statement?

Prove the following equivalences by proving the equivalences of the dual: 9

$\sim (\sim p \wedge q) \vee (\sim p \wedge \sim q) \vee (p \wedge q) \equiv p$

UNIT - II   **18**

2 a. Use Mathematical Induction to prove that $(3^n + 7^n - 2)$ is divisible by 8, for $n \geq 1$ 9

b. Identify the coefficient of;

i) $x^3 y^2 z^3$ in the expansion of $(2x+3y-4z+w)^9$ ii) x^{11} in the expansion of $\left(x^2 - \left(\frac{1}{x} \right) \right)^{10}$ 9

c. Suppose there are six boys and five girls

i) In how many ways can they sit in a row?   9

ii) In how many ways can they sit in a row, if the boys and girls each sit together

iii) In how many ways can they sit in a row, if the girls are to sit together and the boys don't sit together?

iv) How many seating arrangements are there with no two girls sitting together?

UNIT - III   **18**

3 a.   State and prove pigeonhole principle. Show that among $(n + 1)$ positive integers not exceeding 2^n there must be an integer that divides one of the other integers.   9

  b.   Define POSET. Draw the Hasse diagram representing partial order $\{(a, b)|a$ divides $b\}$ on $\{1, 2, 3, 4, 6, 8, 12\}$ and determine whether the relation R is a linear order on set A.   9

  c.   Let $S = \{1, 2, 3\}$ and $A = S \times S$. Define the following relation R on (a, b) R (c, d) if and only if $ad = bc$

  i) Show that r is an equivalence relation   ii) Compute A/R   9

## UNIT - IV   **18**

4 a.   Solve the recurrence relation of the Fibonacci sequence of numbers $f_n = f_{n-1}+f_{n-2}$, $n > 2$ with initial conditions $f_1 = 1$, $f_2 = 1$.   9

  b.   State Inclusion Exclusion Principle. Professor Jane gave DMS class a test consisting of three questions. There are 21 students in her class, and every student answered at least one question. Five students did not answer the first question, seven failed to answer the second question and six did not answer the third question. If nine students answered all questions, how many answered exactly one question?   9

  c.   At a university, seven freshmen, F1, F2, F3, F4, F5, F6, F7, enter the same academic program. Their department head, eager to retain these new students, wants to assign each incoming freshman a mentor from among the upper class men of the program. Seven mentors are chosen, M1, M2, M3, M4, M5, M6 and M7 cannot work with F1 or F5, M4 cannot work with F3 or F6, M5 cannot work with F2 or F7, and M7 cannot work with F4. In how many ways can the department head assign the mentors so that each incoming freshman has a different mentor?   9

UNIT - V   **18**

5 a.   Define Isomorphism of two graphs. Prove that any 2 simple connected graphs with n vertices all of degree 2 are isomorphic.   9

  b.   Define Euler circuit and path. Which of the directed graphs in the below figure have an Euler circuit? Of those that do not, which have an Euler path?   9
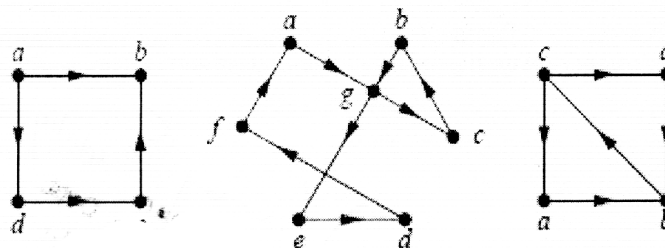
  c.   Define prefix code and balanced tree. If a tree has 5 vertices of degree 2, 3 vertices of degree 3, 4 vertices of degree 4, then how many leaves are there in that tree? Find the optimal code for the message MISSION SUCCESSFUL. Indicate the code.   9

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