



P.E.S. College of Engineering, Mandya - 571 401

(An Autonomous Institution affiliated to VTU, Belagavi)

First Semester, B.E. - Semester End Examination; Dec. 2019

Engineering Mathematics - I

(Common to all Branches)

Time: 3 hrs

Max. Marks: 100

Course Outcome

The Students will be able to:

CO1 - Apply the knowledge of calculus to solve problems related to polar curves and its applications in determining the bentness of a curve.

CO2 - Explain mean value theorems and evaluate the indeterminate form and power series using Taylors and Maclaurin's series.

CO3 - Differentiate the function of several variables differentiate the composite function. Evaluate the vector differentiation.

CO4 - Evaluate some standard integrals by applying reduction formula and solve application problems. Solve differential equations of first order and solve application problems in engineering field.

Note: i) Part - A is compulsory, one question from each unit

ii) Part - B : Answer Two sub-questions for Maximum of 18 marks from each unit.

Q. No.	Questions	Marks	BLs	COs
I : PART - A		10		
I. a.	Find the angle between radius vector and the tangent for the curve $r = a \sin \theta$	2	L1	CO1
b.	State Lagrange's mean value theorem.	2	L1	CO2
c.	Compute U_{xx} if $u = x^3 - 3xy^2 + x + e^x \cos y + 1$	2	L1	CO3
d.	Evaluate $\int_0^{\pi/2} \cos^6 x dx$	2	L1	CO4
e.	Check whether the given differential equation is exact or not: $(y^3 + 3x^2y)dx + (3xy^2 - x^3)dy = 0$	2	L1	CO4
II : PART - B		90		
UNIT - I		18		
1 a. i)	Show that the pairs of curves intersect each other orthogonally, $r^n = a^n \cos n\theta$ and $r^n = b^n \sin n\theta$	9	L1	CO1
ii)	For the equiangular spiral $r = ae^{\theta \cot \alpha}$ where a and α are constants find pedal equation.			
b. i)	Find the radius of curvature for the curve $y = ax^2 + bx + c$ at $x = \frac{1}{2a}(\sqrt{a^2 - 1} - b)$	9	L2	CO1
ii)	Show that for the curve $r^n = a \cos \theta$, e is a constant.			
c.	Find the evolute of the parabola $y^2 = 4ax$ by considering the parametric equations $x = at^2, y = 2at$.	9	L2	CO1
UNIT - II		18		
2 a.	State Cauchy's mean value theorem and verify the same for e^x and e^{-x} in $[3, 7]$	9	L2	CO2
b.	Obtain the Maclaurin's expression of the function $\log(1+x)$ and hence deduce that, $\log \sqrt{\frac{1+x}{1-x}} = x + \frac{x^3}{3} + \frac{x^5}{5} + \dots$	9	L2	CO2

c. Evaluate the following indeterminate forms:

i) $\lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right)^{1/x^2}$

ii) $\lim_{x \rightarrow \frac{\pi}{2}} (2x \tan x - \pi \sec x)$

9 L2 CO2

UNIT - III

18

3 a. i) Apply Euler's theorem to compute the value of $xU_x + yU_y$ for the function

$$u = \sec^{-1} \left(\frac{x^2 + y^2}{x - y} \right).$$

9 L2 CO3

ii) Using the concept of total derivatives, find $\frac{dz}{dt}$ if $z = xy^2 + x^2y$ where $x = at, y = 2at$.

b. i) If $u = f\left(\frac{x}{y}, \frac{y}{z}, \frac{z}{x}\right)$ then by using the definition of composition function

prove that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z} = 0$.

9 L3 CO3

ii) A particle moves along the curve $C : x = t^3, y = t^2 - 2t, z = 10t$ where 't' is the time. Find the component of velocity at $t = 2$ in the direction of $i + 2j + 3k$.

c. Compute a, b and c such that

$\vec{F} = (x + y + az)i + (bx + 2y - z)j + (x + cy + 2z)k$ is irrotational. Also find scalar potential ϕ such that $\vec{F} = \nabla \phi$.

9 L3 CO3

UNIT - IV

18

4 a. Obtain the reduction formula for $\int \sin^n x dx$ and $\int_0^{\pi/2} \sin^n x dx$ where $n = 1, 2, 3, \dots$

9 L2 CO4

hence evaluate $\int_0^{\pi/2} \sin^5 x dx$

b. Apply differentiation under integral sign, evaluate $\int_0^1 \frac{x^\alpha - 1}{\log_e x} dx, \alpha \geq 0$ where α is a

9 L3 CO4

parameter. Hence compute $\int_0^1 \frac{x^3 - 1}{\log x} dx$

c. Tracing the curve $y^2(a - x) = x^3, a > 0$ (cissoid).

9 L3 CO4

UNIT - V

18

5 a. i) Solve: $(x^2 + y^2 + x)dx + xydy = 0$

9 L2 CO4

ii) Solve: $e^y \left(\frac{dy}{dx} + 1 \right) = e^x$

b. i) Show that the orthogonal trajectories of the family of parabolas $y^2 = 4a(x + a)$ forms self orthogonal.

9 L2 CO4

ii) Test for self orthogonality $r^n = a \sin n\theta$

c. If the temperature of the air is 30°C and a metal ball cools from 100°C to 70°C in 15 minutes, find how long will it take for the metal ball to reach a temperature of 40°C.

9 L3 CO4