U.S.N $\square$

# P.E.S. College of Engineering, Mandya - 571401 (An Autonomous Institution affiliated to VTU, Belagavi) <br> Third Semester, B.E. - Semester End Examination; Dec. - 2019 <br> Transform Calculus Fourier series and Numerical Techniques 

(Common to all Branches)
Time: 3 hrs
Max. Marks: 100

## Course Outcomes

The Students will be able to:
CO1: Apply the knowledge of calculus to solve problems related to polar curves and its applications in determining the bentness of a curve.
CO2: Explain mean value theorems and evaluate the indeterminate form and power series using Taylors and Maclaurin's series.
CO3: Differentiate the function of several variables differentiate the composite function. Evaluate the vector differentiation.
CO4: Evaluate some standard integrals by applying reduction formula and solve application problems. Solve differential equations of first order and solve application problems in engineering field.
Note: I) PART - A is compulsory, one question from each unit.
II) PART - B: Answer Two sub-questions for Maximum of 18 marks from each unit.

| Q. No. | Questions |  |  |  |  |  |  | Mark | BLs | COs |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | I : PART - A |  |  |  |  |  |  | 10 |  |  |
| I a. | Write Newton's backward interpolation formula upto fourth degree term. |  |  |  |  |  |  | 2 | L1 | CO1 |
| b. | Write Sterling's formula upto third terms. |  |  |  |  |  |  | 2 | L1 | CO1 |
| c. | Evaluate $\int\left(x+x^{2}\right) \cos n x d x$ |  |  |  |  |  |  | 2 | L1 | CO3 |
| d. | Define Infinite Fourier Transform and inverse Fourier Transform. |  |  |  |  |  |  | 2 | L1 | CO 4 |
| e. | Solve: $\frac{\partial^{2} z}{\partial x \partial t}=e^{-t} \cos x$ |  |  |  |  |  |  | 2 | L1 | CO4 |
|  | II : PART - B |  |  |  |  |  |  | 90 |  |  |
|  | UNIT - I |  |  |  |  |  |  | 18 |  |  |
| 1 a. | From the following table find the number of students who obtained between 40 and 45 marks. |  |  |  |  |  |  | 9 | L2 | CO1 |
|  | Marks | 30-40 | 40-50 | 50-60 | 60-70 | 70-80 |  |  |  |  |
|  | No. of students | 31 | 42 | 51 | 35 | 31 |  |  |  |  |
| b. | Construct the interpolation polynomial for the data given below using Newton's divided difference formula. |  |  |  |  |  |  | 9 | L2 | CO1 |
|  | $x$ 2 |  | 4 | 5 | 6 | 8 | 10 |  |  |  |
|  | $y{ }^{y}$ | 0 | 96 | 196 | 350 | 868 | 1746 |  |  |  |
|  | Hence find the value of $y$ when $x=7$ and $x=9$ |  |  |  |  |  |  |  |  |  |
|  | Contd... 2 |  |  |  |  |  |  |  |  |  |

c. i) Write Gauss's backward interpolation formula up to third degree terms.
ii) Using Stirling's formulae, estimate the value of $\tan \left(16^{\circ}\right)$ and from the data.

| $x$ | 0 | 5 | 10 | 15 | 20 | 25 | 30 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\operatorname{Tan}(x)$ | 0 | 0.0875 | 0.1763 | 0.2679 | 0.3639 | 0.4663 | 0.5774 |

UNIT - II
2 a. i) Write first derivative of Newton's backward formula up to $3^{\text {rd }}$ degree term.
ii) Find the maximum and minimum value of $y$ from the data.
$9 \quad \mathrm{~L} 3 \quad \mathrm{CO} 2$

| $x:$ | -2 | -1 | 0 | 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $y:$ | 2 | -0.25 | 0 | -0.25 | 2 | 15.75 | 56 |

b. i) Write Simpson's $3 / 8^{\text {th }}$ rule for $n=6$
ii) The velocity ' $v$ ' of a particle at distance ' $s$ ' from a point on its path is given by the table:

| $X(f t)$ | 0 | 10 | 20 | 30 | 40 | 50 | 60 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $V\left(f t s^{-1}\right)$ | 47 | 58 | 64 | 65 | 61 | 52 | 38 |

Estimate the time taken to travel 60 ft by using Simpson's $1 / 3^{\text {rd }}$ rule.
c. Evaluate: $\int_{0}^{1} \frac{1}{1+x^{2}} d x$ using Boole's rule for $n=4$ and Weddle's rule for $n=6$.

3 a. Expand the Fourier series of $f(x)=\pi^{2}-x^{2}$ in $-\pi \leq x \leq \pi$ and hence deduce that,
i) $\frac{1}{1^{2}}-\frac{1}{2^{2}}+\frac{1}{3^{2}}-\frac{1}{4^{2}}+---------\infty=\frac{\pi^{2}}{12}$

L2 CO3
ii) $\frac{1}{1^{2}}+\frac{1}{2^{2}}+\frac{1}{3^{2}}+\frac{1}{4^{2}}+---------\infty=\frac{\pi^{2}}{6}$
b. Obtain half range sine and cosine series of $f(x)=\left\{\begin{array}{c}k x, 0 \leq x \leq \frac{l}{2} \\ k(1-x), \frac{l}{2} \leq x \leq l\end{array}\right.$

L3 CO3
c. The following data gives the variations of a periodic current over a period.

| $t$ sec: | 0 | $\mathrm{~T} / 6$ | $\mathrm{~T} / 3$ | $\mathrm{~T} / 2$ | $2 \mathrm{~T} / 3$ | $5 \mathrm{~T} / 6$ | T |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $i$ amp | 1.98 | 1.3 | 1.05 | 1.3 | -0.88 | -0.25 | 1.98 |

L2 CO3
Show that there is a direct current part of 0.75 amp in the variable current and obtain the amplitude of the first and second harmonics.
4 a. If $f(x)=\left\{\begin{array}{c}1-x^{2},|x|<1 \\ 0,|x| \geq 1\end{array}\right.$

## UNIT - IV

Find the Fourier transform of $f(x)$ and hence find the value of
$9 \quad \mathrm{~L} 2 \mathrm{CO} 4$
$\int_{0}^{1} \frac{x \cos x-\sin x}{x^{3}} d x$
b. i) Obtain the Fourier sine transform of the functions,

$$
f(x)=\left\{\begin{array}{cc}
4 x, & 0<x<1 \\
4-x, & 1<x<4 \\
0, & x>4
\end{array}\right.
$$

ii) Find the Fourier cosine transform of $e^{-|x|}$
c. i) State initial value and final value theorems for Z-transform.
ii) Solve difference equation $u_{n+2}+4 u_{n+1}+3 u_{n}=3^{n}$ with $u_{0}=0, u_{1}=1$.

## UNIT - V

5 a. i) Form the PDE by eliminating arbitrary constant in $z=a \log \left(x^{2}+y^{2}\right)+b$
ii) Form the PDE by eliminating arbitrary functions $z=y^{2}+2 f(1 / x+\log y)$
b. i) Define linear PDE.
ii) Solve $\left(z^{2}-2 y z-y^{2}\right) p+(x y+x z) q=x y-x z$

9 L2 CO4
c. Find the various possible solutions of the two dimensional Laplace's equation by the method of separation of variables.

