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P.E.S. College of Engineering, Mandya - 571 401

(An Autonomous Institution affiliated to VTU, Belagavi)

Third Semester, B.E. - Information Science and Engineering
Semester End Examination; March - 2021
Discrete Mathematics and Applications

Time: 3 hrs *Max. Marks: 100*

Course Outcomes

The Students will be able to:

CO1: Verify the correctness of an argument using propositional and predicate logic.

CO2: Demonstrate the ability to solve problems using counting techniques and Combinatorics in the context of discrete probability.

CO3: Solve problems involving recurrence relations.

CO4: Construct proofs using direct proof, proof by contraposition, proof by contradiction, proof by cases, and mathematical induction.

CO5: Ability to Explain and distinguish graphs and their properties.

Note: ***I) PART - A*** *is compulsory.* ***Two*** *marks for each question.*

II) PART - B*: Answer any* ***Two*** *sub questions (from a, b, c) for Maximum of* ***18 marks*** *from each unit.*

Q. No.	Questions	Marks	BLs	COs	POs
	I: PART - A	**10**			
1 a.	Prove that, for any proposition P and Q, the compound proposition is Tautology $\left[(7q)\wedge(p\to q)\right]\to(\neg p)$	2	L5	CO1	PO2
b.	Prove by mathematical induction that for all positive integers $n\geq 1$ $1+2+3+4+....+n=\frac{1}{2}n(n+1)$	2	L5	CO2	PO2
c.	If 5 colors are used to paint 26 doors, prove that atleast 6 doors will have the same colour.	2	L5	CO3	PO3
d.	Find the number of de-arrangements of 1, 2, 3, 4.	2	L3	CO4	PO2
e.	Draw a complete Bi-partite graph of $K_{2,3}$ and $K_{3,3}$.	2	L3	CO5	PO2
	II: PART - B	**90**			
	UNIT - I	**18**			
1 a.	Prove the following logical equivalence without using Truth Table: i) $P\to(q\to r)\Leftrightarrow(p\wedge q)\to r$ ii) $\left[\neg p\wedge(\neg q\wedge r)\right]\vee(q\wedge r)\vee(p\wedge r)\Leftrightarrow r$	9	L5	CO1	PO4
b.	Test the validity of the following argument i) $(\neg p\vee\neg q)\to(r\wedge s)$ $r\to t$ $\neg t$ ________ $\therefore p$ ii) $(\neg p\vee q)\to r$ $r\to(s\vee t)$ $\neg s\wedge\neg u$ $\neg u\to\neg t$ ________ $\therefore p$	9	L5	CO1	PO4

Contd... 2

c. Find whether the following argument is valid.

No engineering student of first or second semester studies logic

Anil is an Engineering student who studies logic

$\therefore$ Anil is not in second semester

9 L5 CO1 PO4

UNIT - II 18

2 a. Show that $2^n > n^2$ for all positive integers n greater than 4. 9 L5 CO2 PO3

b. i) A student taking a history examination is directed to answer any seven of 10 essay questions. There is no concern about order here, find the number of ways the student can answer the examination.

ii) If the student must answer three question from the first five and four questions from the last five. By the rule of product, find the number of ways the student can complete the examination. 9 L5 CO2 PO4

iii) At Rydell High school, the gym teacher must select nine girls from the junior and senior classes for a volleyball team. If there are 28 juniors and 25 seniors, find the number of ways, this selection can be made.

c. Prove that, for positive integer n, t, the coefficient of $x_1^{n1} x_2^{n2} x_3^{n_3} \ldots x_t^{n_t}$ in the expansion of $\left(x_1 + x_2 + x_3 + \ldots x_i\right)^n$ is $\dfrac{n!}{n_1!n_2!n_3!\ldots n_t!}$ 9 L5 CO2 PO3

Where each n_i is an integer with $0 \le n_i \le n$, for all $1 \le i \le t$ and

$n_1 + n_2 + n_3 + \ldots + n_t = n$

UNIT - III 18

3 a. I) Let *f: A→B* and *g: B→C* be any two functions. Prove the following are True: 6

i) If *f* and *g* are one-to-one, So is *gof*

ii) If *gof* is one-to-one, then *f* is one-to one L3 CO3 PO3

II) Let *f* and *g* be functions from *R* to *R* defined by f(x) = ax + b and *g(x) = cx+ d*. What relationship must be satisfied by *a, b, c, d* if *gof = fog* 3

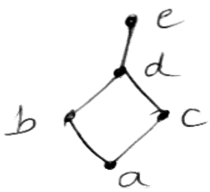
b. Let *A* = <1, 2, 3, 4, 5} define a relation *R* on *A x A* by $(x_1, y_1)\ R\ (x_2, y_2)$ if and only if $x_1 + y_1 = x_2 + y_2$

i) Verify that *R* is an equivalence relation on *A x A* 9 L3 CO3 PO4

ii) Determine the equivalence classes [(1, 3) (2, 4)] and [(1, 1)]

iii) Determine the partition of *A x A* induced by *R*

c. For A = {a, b, c, d, e} the Hasse diagram for the poset (A, R) is as shown below



9 L3 CO3 PO3

i) Determine the relation matrix for *R* ii) Construct the digraph for *R*

UNIT - IV **18**

4 a. Determine the number of positive integers x where $x \leq 9{,}999{,}999$ and the sum of the digits in x equal to 31. 9 L5 CO4 PO3

b. Find the rook polynomial for the board made up of the shaded square in the below figure 9 L5 CO4 PO3

	1	2		
3		4		
	5		6	7
			8	

c. Solve the Recurrence Relation $F_{n+2} = F_{n+1} + F_n$ for $n \geq 0$. Given $F_0 = 0,\ F_1 = 1$. 9 L3 CO4 PO4

UNIT - V **18**

5 a. Define Isomorphism of two graphs. Prove that any simple connected graph with n vertices all of degree 2 are isomorphic. 9 L3 CO5 PO2

b. Construct an optimal prefix code for the message MISSION SUCCESSFUL. Indicate the code for the message. 9 L5 CO5 PO4

c. Define with example: 9 L2 CO5 PO1

i) Rooted Tree

ii) Binary tree

iii) Isolated Vertex

iv) Pendant Vertex

v) Regular Graph

vi) Bipartite Graph

vii) Complete Bipartite graph

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