

**P.E.S. College of Engineering, Mandya - 571 401***(An Autonomous Institution affiliated to VTU, Belagavi)***First Semester, B.E. - Semester End Examination; April - 2021****Engineering Mathematics - I****(Common to all Branches)***Time: 3 hrs**Max. Marks: 100***Course Outcomes***The Students will be able to:**CO1: Apply the knowledge of calculus to solve problems related to polar curves and its applications in determining the bentness of a curve.**CO2: Explain mean value theorems and evaluate the indeterminate form and power series using Taylors and Maclaurin's series.**CO3: Differentiate the function of several variables differentiate the composite function. Evaluate the vector differentiation.**CO4: Evaluate some standard integrals by applying reduction formula and solve application problems. Solve differential equations of first order and solve application problems in engineering field.***Note:** *I) PART - A is compulsory. Two marks for each question.**II) PART - B: Answer any Two sub questions (from a, b, c) for Maximum of 18 marks from each unit.*

Q. No.	Questions	Marks	BLs	COs	POs
I : PART - A		10			
I a.	Find the angle between radius vector and the tangent for the curve $r = a(1 - \cos \theta)$.	2	L1	CO1	PO1
b.	Evaluate $\lim_{x \rightarrow 0} x^2 \log x$.	2	L1	CO2	PO1
c.	Find $\text{Div } \vec{F}$, where $\vec{F} = \nabla (x^3 + y^3 + z^3 - 3xyz)$.	2	L1	CO3	PO1
d.	Evaluate $\int_0^{\pi/2} \sin^6 x \cos^2 x dx$.	2	L1	CO4	PO1
e.	Solve: $(y^3 - 3x^2y)dx - (x^3 - 3xy^2)dy = 0$.	2	L1	CO4	PO1
II : PART - B		90			
UNIT - I		18			
1 a.	i) Find the pedal equation of $\frac{2a}{r} = (1 + \cos \theta)$.	9	L2	CO1	PO2
	ii) Show that the curves $r^n = a^n \cos n\theta$ and $r^n = b^n \sin n\theta$ intersect each other orthogonally.				
b.	Find the radius of curvature for the curve $y^2 = \frac{4a^2(2a-x)}{x}$, where the curve meets the x-axis.	9	L3	CO1	PO2
c.	Find the evolute of the parabola $y^2 = 4ax$.	9	L3	CO1	PO2
UNIT - II		18			
2 a.	i) State Lagrange's mean value theorem.	9	L2	CO2	PO2
	ii) Verify Cauchy's mean value theorem for the functions, e^x and e^{-x} in $[a, b]$.				

- b. i) State Taylor's series of $f(x)$ about $x=a$ upto fourth degree term.

9 L2 CO2 PO2

- ii) Obtain the Maclaurin's expansion of $\log(1+\sin x)$.

- c. Evaluate; i) $\lim_{x \rightarrow 1} \left[\frac{x}{x-1} - \frac{1}{\log x} \right]$ ii) $\lim_{x \rightarrow 0} \left(\frac{\tan x}{x} \right)^{1/x}$

9 L2 CO2 PO2

UNIT - III**18**

- 3 a. If $u = \tan^{-1} \left(\frac{x^3 + y^3}{x - y} \right)$ show that,

9 L2 CO3 PO2

i) $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \sin 2u$ ii) $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = \sin 4u - \sin 2u$

- b. Write the total derivative rule.

If $u = f(x - y, y - z, z - x)$ show that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$.

9 L3 CO3 PO1

- c. i) Find the directional derivative of $\phi = x^2 yz + 4xz^2$ at $(1, -2, -1)$ in the direction of $2i - j - k$.

9 L3 CO3 PO2

- ii) Show that the vector field,

$$\vec{F} = (x^2 - yz)i + (y^2 - zx)j + (z^2 - xy)k \text{ is irrotational.}$$

UNIT - IV**18**

- 4 a. Obtain the reduction formula for $\int \sin^n x dx$ and hence $\int_0^{\pi/2} \sin^n x dx$, where n is a positive integer.

9 L2 CO4 PO2

- b. Evaluate $\int_0^{\infty} e^{-ax} \frac{\sin x}{x} dx$ and hence evaluate $\int_0^{\infty} \frac{\sin x}{x} dx$.

9 L3 CO4 PO2

- c. Trace the curve $r = a(1 + \cos \theta)$ (cardioid).

9 L3 CO4 PO2

UNIT - V**18**

- 5 a. i) Solve: $\left(1 + e^{x/y}\right)dx + e^{x/y} \left(1 - \frac{x}{y}\right)dy = 0$.

9 L2 CO4 PO1

- ii) Solve: $y(xy + 1)dx - x(xy - 1)dy = 0$.

- b. i) Solve: $\frac{dy}{dx} + \frac{y}{x} = y^2 x$.

9 L2 CO4 PO2

- ii) Solve: $(xy^3 + y)dx + 2(x^2 y^2 + x + y^4)dy = 0$.

- c. If the temperature of the air is 30°C and a metal ball cools from 100°C to 70°C in 15 minutes, find how long will it take for the metal ball to reach a temperature of 40°C ?

9 L3 CO4 PO2