**U.S.N** | | | | | | | | | |

# P.E.S. College of Engineering, Mandya - 571 401

**(An Autonomous Institution affiliated to VTU, Belagavi)**

## Third Semester, B.E. - Semester End Examination; March - 2021

## Transform Calculus, Fourier Series and Numerical Techniques

(Common to all Branches)

*Time: 3 hrs* *Max. Marks: 100*

***Course Outcomes***

*The Students will be able to:*

*CO1: Apply forward, backward difference formulae and central differences formulae in solving interpolation-extrapolation problems in engineering field.*

*CO2: Numerical differentiation and integration rules in solving engineering where the handlings of numerical methods are inevitable.*

*CO3: Apply the knowledge of periodic function, Fourier series, complex Fourier series, Fourier sine/cosine series of a function valid in different periods. Analyze engineering problems arising in control theory/fluid flow phenomena using harmonic analysis.*

*CO4: Understand complex/infinite Fourier transforms Fourier sine and Fourier cosine transforms with related properties. Analyze the engineering problems arising in signals and systems, digital signal processing using Fourier transform techniques. Define Z-transforms& find Z-transforms of standard functions to solve the specific problems by using properties of Z-transforms. Identify and solve difference equations arising in engineering applications using inverse Z-transforms techniques.*

*CO5: Define Partial Differential Equations (PDE's), order, degree and formation of PDE's and, to solve PDE's by various methods of solution. Explain one - dimensional wave and heat equation and Laplace's equation and physical significance of their solutions to the problems selected from engineering field.*

***Note:*** ***I) PART - A*** *is compulsory.* ***Two*** *marks for each question.*

***II) PART - B****: Answer any* ***Two*** *sub questions (from a, b, c) for Maximum of* ***18 marks*** *from each unit.*

**Q. No. Questions Marks BLs COs POs**

**I : PART - A 10**

I a. Construct the Newton's Backward difference table for the data given below, 2 L1 CO1 PO1

| $X$ | 2 | 4 | 6 | 8 |
|---|---|---|---|---|
| $Y$ | 10 | 96 | 196 | 350 |

b. Write the first derivative Newton's forward formula up to third degree. 2 L1 CO2 PO1

c. Evaluate: $\int e^{ax} \cos bxdx$. 2 L1 CO3 PO1

d. Define Z-Transform of $u_n$. 2 L1 CO4 PO1

e. Solve by direct integration $\frac{\partial^2 z}{\partial x\,\partial y} = \cos x \cos y$. 2 L1 CO4 PO1

**II : PART - B 90**

**UNIT - I 18**

1 a. i) Define Extrapolation.

ii) A survey conducted in a slum locality reveals the following information as classified below. 9 L2 CO1 PO1

| Income per day (Rs.) | Under 10 | 10 - 20 | 20 - 30 | 30 - 40 | 40 - 50 |
|---|---|---|---|---|---|
| Number of persons | 20 | 45 | 115 | 210 | 115 |

Estimate the probable number of person in the income group 20 to 25.

b. i) Write a Lagrange's inverse interpolation formula for $x = f(y)$.

ii) The following table gives the normal weights of babies during first eight months of life. 9 L3 CO1 PO1

| Age (months) | 0 | 2 | 5 | 8 |
|---|---|---|---|---|
| Weight (pounds) | 6 | 10 | 12 | 16 |

Estimate the weight of the baby at the age of seven months using Lagrange's interpolation formula.

c. i) Write Gauss's forward interpolation formula up to 4th degree terms.

ii) Use Stirling's formula to compute $u_{14.2}$ from the following: 9 L3 CO1 PO2

$u_{10} = 0.240,\ u_{12} = 0.281,\ u_{14} = 0.318,\ u_{16} = 0.352,\ u_{18} = 0.384$

### UNIT - II
18

2 a. Find the maximum and minimum values of the function $y = f(x)$ from the following data. 9 L1 CO2 PO1

| $x$ | 1 | 3 | 5 | 7 | 9 |
|---|---|---|---|---|---|
| $y$ | 9 | 11 | 13 | 63 | 209 |

b. i) Write the Trapezoidal rule for $n = 6$.

ii) Use Simpson's $\left(\frac{1}{3}\right)^{rd}$ rule to obtain the approximate value of $\int_0^{0.6} e^{-x^2}dx$ by considering 6 equal strips. 9 L3 CO2 PO2

c. i) Write Boole's rule for $n = 8$.

ii) Evaluate $\int_0^1 \frac{x}{1+x^2}dx$, by Weddle's rule taking seven ordinates and hence find $\log_e 2$. 9 L3 CO2 PO2

### UNIT - III
18

3 a. Obtain the Fourier series for the function $f(x) = x - x^2$ in $(-\pi, \pi)$ and hence deduce that $\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - ..........$ 9 L3 CO3 PO2

b. i) Obtain the complex form of the Fourier series for the function.

$$f(x) = \begin{cases} -k & \text{in } -\pi < x < 0 \\ k & \text{in } \ 0 < x < \pi \end{cases}$$

9 L3 CO3 PO2

ii) Expand $f(x) = 2x - 1$ as the cosine half range Fourier series in $0 < x < 1$.

c. Express $y$ as a Fourier series up to the second harmonic given the following data. 9 L2 CO3 PO2

| $x$ | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| $y$ | 4 | 8 | 15 | 7 | 6 | 2 |

## UNIT - IV

**18**

4 a. Find the Fourier Transform of $f(x)=\begin{cases}1-|x| & \text{in } |x|\le 1\\ 0 & \text{in } |x|>1\end{cases}$ and hence evaluate $\int_0^\infty \frac{\sin^2 t}{t^2}dt.$ 9 L2 CO4 PO2

b. Solve the integral equation $\int_0^\infty f(\theta)\cos\alpha\theta d\theta=\begin{cases}1-a & 0\le\alpha\le 1\\ 0 & \alpha>1\end{cases}$ and hence evaluate, $\int_0^\infty \frac{\sin^2 t}{t^2}dt$. 9 L3 CO4 PO2

c. i) Find the Z-transform of $(n+1)^2$.

ii) Solve by using Z-Transforms: $y_{n+1}+\frac{1}{4}y_n=\left(\frac{1}{4}\right)^n,\ y_0=0.$ 9 L3 CO4 PO2

## UNIT - V

**18**

5 a. i) Form the Partial differential equation by eliminating the arbitrary constants for $2z=\frac{x^2}{a^2}+\frac{y^2}{b^2}$.

ii) Form the Partial differential equation by eliminating the arbitrary function for $\varphi\left(xy+z^2,\ x+y+z\right)=0.$ 9 L1 CO4 PO1

b. i) Define homogeneous partial differential equation.

ii) Solve: $x\left(y^2-z^2\right)p+y\left(z^2-x^2\right)q=z\left(x^2-y^2\right).$ 9 L3 CO4 PO2

c. Obtain the various possible solutions of the two dimensional Laplace equations $u_{xx}+u_{yy}=0,$ by the method of separation of variables. 9 L3 CO4 PO2

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