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# P.E.S. College of Engineering, Mandya - 571401 <br> (An Autonomous Institution affiliated to VTU, Belagavi) <br> First Semester, M.Tech. - Mechanical Engineering (MMDN) <br> Semester End Examination; Jan. - 2020 <br> Theory of Elasticity 

Time: 3 hrs
Max. Marks: 100
Note: i) Answer FIVE full questions, selecting ONE full question from each unit.
ii) Missing data, if any, may be suitably assumed.

UNIT - I
1 a. Derive the expression for normal stress acting on an arbitrary plane.
b. At a point in a material, a resultant stress of 151 MPa acts in a direction, making angles of $52^{\circ}$, $79^{\circ}$ and $40.13^{\circ}$ with coordinate axes $x, y$ and $z$ respectively. Find the normal and shear stresses on the plane whose normal makes angles of $33^{\circ}, 65^{\circ}$ and $70^{\circ}$ respectively with $x, y$ and $z$ axes.

2 a. Obtain the expression for stress induced in a body subjected to uniform temperature rise and the body is prevented from having any displacements.
b. A rectangular bar of cross section $30 \mathrm{~mm} \times 25 \mathrm{~mm}$ is subjected to an axial tensile force of 180 kN . Determine the normal, shear and resultant stresses on a plane, whose normal has direction cosines $l=m=n=\frac{1}{\sqrt{3}}$.

UNIT - II
3. Derive the expression for change in length of a linear element.

4 a. Displacement field imposed on a body is given by $U=\left(x y i+3 x^{2} j+4 k\right) 10^{-2}$. A line PQ in the body has direction cosines $l, m$ and $n$ equal to $0.2,0.8$ and 0.555 respectively. If ' $P$ ' has coordinates $(2,1,3)$ and $\mathrm{PQ}=\Delta \mathrm{S}$. Find $\mathrm{P}^{\prime} \mathrm{Q}^{\prime}$ after deformation.
b. Displacement field for a body is given by,

$$
U=A\left(x^{2}+y\right) i+A(y+z) j+A\left(x^{2}+2 z^{2}\right) k
$$

Where $\mathrm{A}=10^{-3}$. At a point $\mathrm{P}(2,2,3)$ two lines PQ and PR are considered with direction cosines $l_{1}=m_{1}=n_{1}=\frac{1}{\sqrt{3}}$ and $l_{2}=m_{2}=\frac{1}{\sqrt{2}}, n_{2}=0$ respectively. Determine the angle between the lines before and after deformation.

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UNIT - III
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5 a. Explain Saint-Venant's Principle.
b. A cubical element is subjected to the following state of stress $\sigma_{x}=100 \mathrm{MPa}, \sigma_{y}=-20 \mathrm{MPa}$, $\sigma_{\mathrm{z}}=-40 \mathrm{MPa}, \tau_{\mathrm{xy}}=\tau_{\mathrm{yz}}=\tau_{\mathrm{zx}}=0$. Assuming the material to be homogeneous and isotropic, determine the principal shear strains and octahedral shear strain, if $\mathrm{E}=200 \mathrm{GPa}$, and $\gamma=0.25$.

6 a. Explain principle of superposition.
b. The state of strain at a point is given by $\epsilon_{x}=0.001, \epsilon_{y}=-0.003, \epsilon_{t}=\gamma_{x y}=0, \gamma_{x z}=-0.004$, $\gamma_{y z}=0.001$. Determine the stress tensor at this point. Also find Lame's constant.
$\mathrm{E}=210 \mathrm{GPa}, \gamma=0.28$.

## UNIT - IV

7 a. State and prove Maxwell reciprocal theorem.
b. Determine the support reaction for the propped cantilever shown in Fig. Q7(b).


Fig. Q 7(b)
8 a. Explain Kirchhoff's theorem.
b. Three elastic members AB, BD and CD are connected by smooth pins as shown in Fig. Q8(b). All the members have same cross sectional area and are of same material. BD is 1 m long and members $A D$ and CD are 2 m long each. Determine the deflection of ' $D$ ' under load W , if cross sectional area of each member is $100 \mathrm{~mm}^{2} . \mathrm{E}=200 \mathrm{GPa}$ and load $\mathrm{W}=10 \mathrm{kN}$. Use principle of virtual work.


Fig. Q 8(b)
UNIT - V
9. Assuming a fourth degree polynomial function of,
$\phi=\frac{A}{24} x^{4}+\frac{B}{6} x^{3} y+\frac{C}{2} x^{2} y^{2}+\frac{D}{6} x y^{3}+\frac{E}{24} y^{4}$
Obtain the expressions for $\sigma_{\mathrm{x}}, \sigma_{\mathrm{y}}$ and $\tau_{\mathrm{xy}}$ for a narrow cantilever beam subjected to end load.
10. For a plane stress case, obtain the expressions for stresses and deformation, for a thick walled cylinder subjected to internal and external pressure.

