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P.E.S. College of Engineering, Mandya - 571 401

(An Autonomous Institution affiliated to VTU, Belagavi)

Eighth Semester, B.E. - Semester End Examination; Aug. / Sep. - 2020

Linear Algebra and Analysis

Time: 3 hrs Max. Marks: 100

Note: i) Answer ***TWO*** *full questions, selecting* ***ONE*** *full question from* ***UNIT - I*** *and* ***UNIT - II.***
ii) Answer any ***THREE*** *full questions, choosing from* ***UNIT - III, UNIT - IV*** *and* ***UNIT - V.***

UNIT - I

1 a. Show that for any square matrix A, the matrix $A - A^T$ is skew-symmetric matrix. Express $\begin{bmatrix} 3 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & -1 & 3 \end{bmatrix}$ as a sum of symmetric and skew-symmetric matrix. 6

b. i) Define a Skew-Hermitian matrix with an example.

ii) Show that $A = \begin{bmatrix} 1 & 2+i & 3i & i \\ 2-i & 5 & 1-i & 2i \\ -3i & 1+i & 6 & 7+i \\ -i & -2i & 7-i & 8 \end{bmatrix}$ is Hermitian matrix. 7

c. Express $A = \begin{bmatrix} -1 & 2+i & 5-3i \\ 2-i & 7 & 5i \\ 5+3i & -5i & 2 \end{bmatrix}$ in the form $P + iQ$ where P is real symmetric and Q are real skew symmetric matrix. 7

OR

2 a. Define involutory and nilpotent matrices. Show that, $A = \begin{bmatrix} -5 & -8 & 0 \\ 3 & 5 & 0 \\ 1 & 2 & -1 \end{bmatrix}$ is an involutory matrix. 6

b. Show that,

$C = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 5 & 0 & 0 & 0 \\ 7 & 8 & 0 & 0 \\ 11 & 2 & 9 & 0 \end{bmatrix}$ is a nilpotent matrix and find its degree. What is the determinant of C? 7

c. Define real orthogonal and normal matrices with examples. Prove that the matrix $A = \begin{bmatrix} \frac{1}{9} & \frac{8}{9} & -\frac{4}{9} \\ \frac{4}{9} & -\frac{4}{9} & -\frac{7}{9} \\ \frac{8}{9} & \frac{1}{9} & \frac{4}{9} \end{bmatrix}$ is orthogonal. 7

UNIT - II

3 a. Reduce the matrix A to its normal form $A=\begin{bmatrix}1 & 2 & 0 & -1\\ 3 & 4 & 1 & 2\\ -2 & 3 & 2 & 5\end{bmatrix}$. 6

b. Find the inverse of the matrix using elementary matrices $A=\begin{bmatrix}1 & 0 & 2\\ 2 & -1 & 3\\ 4 & 1 & 8\end{bmatrix}$. 7

c. Find A^{-1} by Cayley-Hamilton theorem, if, $A=\begin{bmatrix}1 & 2 & 4\\ -1 & 0 & 3\\ 3 & 1 & -2\end{bmatrix}$. 7

OR

4 a. Define a matrix polynomial. If $A=\begin{bmatrix}1 & -2\\ 4 & 5\end{bmatrix}$, then determine $f(A)$ where $f(t)=t^2-3t+7$ and $f(t)=t^2-6t+13$. 6

b. Define characteristic and minimal polynomial of a matrix. Find the characteristic polynomial $\Delta(t)$ of $\begin{bmatrix}6 & -3 & -2\\ 4 & -1 & -2\\ 10 & -5 & -3\end{bmatrix}$. 7

c. Show that similar matrices have same minimal polynomial. Find the minimal polynomial $m(t)$ of $\begin{bmatrix}2 & 2 & -5\\ 3 & 7 & -15\\ 1 & 2 & -4\end{bmatrix}$. 7

UNIT - III

5 a. Define; i) Vector space and ii) Subspace with suitable examples. 6

b. i) Define linear dependence of vectors.

ii) Show that $u=(1,2,3), v(2,5,7), w=(1,3,5)$ are linearly independent. 7

c. Define basis of a vector space. Verify whether the set $\{(1,2,1),(2,1,0),(1,-1,2)\}$ a basis for the vector space $R^3(R)$. 7

6 a. Find the coordinate vector of $A=\begin{bmatrix}3 & -5\\ 6 & 7\end{bmatrix}$ in the real vector space $M=M_{2,2}$ relative to the basis 6

i) $S=\left\{\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix},\begin{bmatrix}1 & -1\\ 1 & 0\end{bmatrix},\begin{bmatrix}1 & -1\\ 0 & 0\end{bmatrix},\begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix}\right\}$ ii) $S=\left\{\begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix},\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix},\begin{bmatrix}0 & 0\\ 1 & 0\end{bmatrix},\begin{bmatrix}0 & 0\\ 0 & 1\end{bmatrix}\right\}$

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b. i) Define a linear transformation.

ii) Find the matrix of linear transformation $T: R^3 \to R^3$ defined by $T(x, y, z) = (2y + z, x - 4y, 3x)$ with respect to basis $S = \{(1,1,1)(1,1,0)(1,0,0)\}$ 7

c. Let: $T: R^3 \to R^3$ be the linear mapping defined by $T(x, y, z) = (x + 2y - z, y + z, x + y - 2z)$. Find the rank and nullity of T. 7

UNIT - IV

7 a. Define a convergent series. Show that the series $\sum_{n=1}^{\infty}\left[\sqrt{n+2} - \sqrt{n+1}\right]$ is divergent. 6

b. Discuss the nature of the series $\frac{1}{4.7.10} + \frac{4}{7.10.13} + \frac{9}{10.13.13} + \frac{16}{13.16.19} + \ldots$ 7

c. Test for the convergence of the series $\frac{3}{4} + \frac{3.4}{4.6} + \frac{3.4.5}{3.5.7} + \ldots$ 7

8 a. Test for the convergence of the series $\frac{1}{2}x + \frac{1.3}{2.4}x^2 + \frac{1.3.5}{2.4.6}x^2 + \ldots$ 6

b. Discuss the convergence of $\sum_{n=1}^{\infty} \frac{(n+1)^n x^n}{n^{n+1}}$. 7

c. Determine the radius and circle of convergence of the series $x + \frac{x^2}{2^2} + \frac{x^3}{3^2} + \frac{x^4}{4^2} + \ldots.$ 7

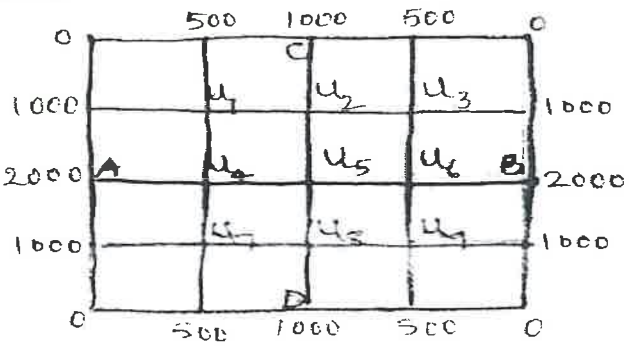
UNIT - V

9 a. Solve numerically $\frac{\partial^2 u}{\partial x^2} = 0.0625 \frac{\partial^2 u}{\partial t^2}$ subject to $u = (0,t) = 0, u(5,t) = 0, u_t(x,0) = x^2(x-5)$ by taking $h = 1$, for $0 \le t \le 1$. 10

b. Solve $u_t = u_{xx}$ subject to the conditions $u(0,t) = 0$, $u(1,t) = 0$, $u(x,0) = \sin(\pi x)$ for $0 \le t \le 0.1$ by taking $h = 0.2$. Write down the following values from the table; 10

i) u(0.2, 0.04) *ii) u(0.4, 0.08)* *iii) u(0.6, 0.06)*

10 a. Solve the elliptic equation $u_{xx} + u_{yy} = 0$ for the square mesh of the following figure with boundary values as shown. 10



b. Solve $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ in $0 < x < 5$, $t \ge 0$ given that $u(x, 0) = 20$, $u(0,t) = 0$, $u(5,t) = 100$. Compute u for time-step with $h = 1$ by Crank-Nicholson method. 10

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