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P.E.S. College of Engineering, Mandya - 571 401

(An Autonomous Institution affiliated to VTU, Belagavi)

Eighth Semester, B.E. - Semester End Examination; July - 2021

Linear Algebra and Analysis

Time: 3 hrs Max. Marks: 100

Note: *Answer any **FIVE** full questions.*

1 a. i) For any symmetric matrix A, prove that A^2 is symmetric.

ii) Express $A=\begin{bmatrix}1&2&3\\4&5&6\\7&0&0\end{bmatrix}$ as a sum of symmetric and skew symmetric matrix. 6

b. i) For any square matrix A, show that $A+A^T$ is symmetric.

ii) If $A=\begin{bmatrix}-1&2+i&5-3i\\2-i&7&5i\\5+3i&-5i&2\end{bmatrix}$ show that iA is skew Hermitian. 7

c. i) Define unitary matrix with an example.

ii) Express $\begin{bmatrix}3&3+2i&0\\3-2i&0&1+3i\\0&1-3i&-1\end{bmatrix}$ in the form $P+iQ$ where P is a real symmetric and Q is a real skew symmetric matrix. 7

2 a. Define a nilpotent matrix and verify whether, $A=\begin{bmatrix}0&1&0\\0&0&1\\0&0&0\end{bmatrix}$ is nilpotent. 6

b. i) Define an orthogonal matrix with an example.

ii) Show that the matrix $A=\begin{bmatrix}3&-4&4\\0&-1&0\\-2&2&-3\end{bmatrix}$ is involutory 7

c. Show that $A=\begin{bmatrix}\frac{1}{9}&\frac{8}{9}&-\frac{4}{9}\\\frac{4}{9}&-\frac{4}{9}&-\frac{7}{9}\\\frac{8}{9}&\frac{1}{9}&\frac{4}{9}\end{bmatrix}$ is orthogonal 7

3 a. Reduce the matrix $A=\begin{bmatrix}1&2&-1&3\\4&1&2&1\\3&-1&1&2\\1&2&0&1\end{bmatrix}$ to normal form. 6

b. Find the inverse of $A=\begin{bmatrix}1&2&-4\\-1&-1&5\\2&7&-3\end{bmatrix}$ using elementary matrices 7

c. Find A^{-1} by Cayley-Hamilton theorem if $A = \begin{bmatrix} 2 & 1 & 1 \\ -1 & -2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$. 7

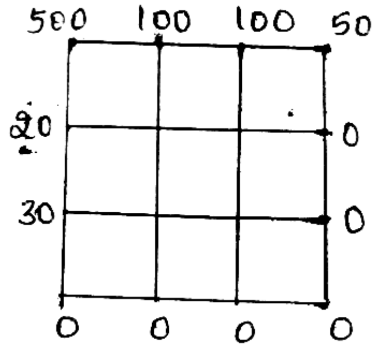
4 a. Determine the characteristic polynomial of $\begin{bmatrix} 9 & -1 & 5 & 7 \\ 8 & 3 & 2 & -4 \\ 0 & 0 & 3 & 6 \\ 0 & 0 & -1 & 8 \end{bmatrix}$. 6

b. Define minimal polynomial and determine the same for $\begin{bmatrix} 2 & 2 & -5 \\ 3 & 7 & -15 \\ 1 & 2 & -4 \end{bmatrix}$. 7

c. Show that $A = \begin{bmatrix} 1 & 3 \\ 4 & 5 \end{bmatrix}$ satisfies its characteristic polynomial. 7

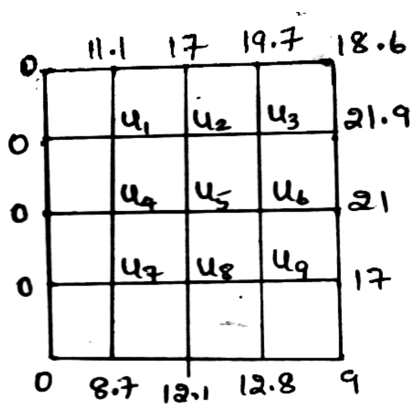
5 a. Solve numerically $u_{xx} = 0.0625\, u_{tt}$ subjected to $u(0, t) = 0$, $u(5, t) = 0$, $u_t(x, 0) = 0$ and $u(x, 0) = x^2(x-5)$ by taking $h = 1$ for $0 \le t \le 1$. 10

b. Solve $u_{xx}+u_{yy}= 0$ in the following square region with the boundary conditions as indicated in the figure. 10



6 a. Find the numerical solution of the parabolic equation $\frac{\partial^2 y}{\partial x^2} = 2\frac{\partial u}{\partial t}$ when $u(0, t) = 0 = u(4,t)$ and $u(x,0) = x(4-x)$ by taking $h = 1$. Find the values upto $t = 5$. 10

b. Solve Laplace equation $u_{xx} + u_{yy}=0$ for the following square mesh with boundary values as shown in the following figure. 10



7 a. Illustrate convergence and divergence of an infinite series with examples. 6

b. State comparison test and apply the same to determine the convergence of $V=\sum_{n=1}^{\infty}\left[\sqrt{n^4+1}-\sqrt{n^4-1}\right].$ 7

c. Verify the convergence of $\frac{2}{3+4}+\frac{2^2}{3^2+4}+\frac{2^3}{3^3+4}+\cdots$ 7

8 a. Discuss the convergence of the series $1+\frac{2}{3}x+\left(\frac{3}{4}\right)^2x^2+\left(\frac{4}{5}\right)^3x^3+\cdots$ 6

b. Discuss the convergence of $\frac{1^2}{4^2}+\frac{1^2.5^2}{4^2.8^2}+\frac{1^2.5^2.9^2}{4^2.8^2.12^2}+\cdots$ 7

c. Find the radius of convergence of the followings: 7

$i)\sum_{n=1}^{\infty}\left(1+\frac{1}{n}\right)^{n^2}.x^n$ $\qquad ii)\sum_{n=1}^{\infty}\frac{x^n}{n!}$

9 a. Show that R^2 is a vector space over R. 6

b. i) Define linearly dependent and independent vectors. 7

ii) Show that the vectors (1, 2, –1) , (2, 2, 1), (1, –2, 3) are linearly independent in R^3

c. i) Define linear combination of vectors 7

ii) Show that {(1, 2, 1), (1, 0, –1), (0, –3, 2)} forms a basis of $R^3(R)$

10 a. Find the basis and dimension of the subspace $w=\{(x_1,x_2\cdots x_n)\,|\,x_1=x_n\}$ *of* R^n. 6

b. Find the rank of T, *given* $T:R^4\to R^3$ is a linear map defined by, 7

$T(x,\,y,\,z,\,t)=(x-y+z+t,\,2x-2y+3z+4t,\,3x-3y+4z+5t).$

c. Find the matrix of a linear transformation $T:R^2\to R^2$ by $T(x,\,y)=(2x+3y,\,4x-5y)$ relative to the basis {(1, 2), (2, 5)}. 7

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