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P.E.S. College of Engineering, Mandya - 571 401

(An Autonomous Institution affiliated to VTU, Belagavi)

First Semester, M. Tech - Civil Engineering (MCAD)

Semester End Examination; April / July - 2021

Continuum Mechanics - Classical and FF Approach

Time: 3 hrs *Max. Marks: 100*

***Note**: Answer **FIVE** full questions, selecting **ONE** full question from each unit.*

UNIT - I

1 a. Derive the expression for the equilibrium of plane element in polar coordinate. 10

b. A rectangular bar of metal of cross section 30 mm × 25 mm is subjected to an axial tensile force of 180 kN. Calculate the normal, shear and resultant stresses on a plane whose normal has the following direction cosines: 10

i) $l = m = \frac{1}{\sqrt{2}}$ and $n = 0$ ii) $l = m = n = \frac{1}{\sqrt{3}}$

2. Given stress at a point as follows:

$$T = \begin{bmatrix} 10 & 4 & 8 \\ 4 & 20 & -6 \\ 8 & -6 & -30 \end{bmatrix} MPa$$ 20

Find principal stresses, maximum shear stresses, octahedral stresses with respect to principal stresses and plane of intermediate principal stress.

UNIT - II

3 a. Derive the strain displacement relation for a 2D element in Cartesian coordinate system. 10

b. Under what conditions are the following expressions for the components of strain at a point compatible? 10

$\epsilon_x = 2axy^2 + by^2 + 2cxy$

$\epsilon_y = ax^2 + bx$

$\gamma_{xy} = \alpha x^2 y + \beta xy + \phi x^2 + \eta y$

4 a. Derive the equations for principal strain in 3D coordinate system and hence obtain the equations for strain invariants. 10

b. The strain tensor at a point in a body is given by, 10

$$\epsilon = \begin{bmatrix} 0.0001 & 0.0002 & 0.0005 \\ 0.0002 & 0.0003 & 0.0004 \\ 0.0005 & 0.0004 & 0.0005 \end{bmatrix}$$

Determine; i) Octahedral normal and shearing strains

ii) Deviatoric and spherical strain tensors

Contd… 2

UNIT - III

5 a. Define generalized Hook's law. Obtain the expressions for stress strain relationships in a three dimensional problem in Cartesian coordinate system. 10

b. The state of strain at a point is given by,

$\in_x = 0.001$ $\gamma_{yz} = 0.001$

$\in_y = -0.003$ $\gamma_{xz} = -0.004$ $\in_z = \gamma_{xy} = 0$ 10

Determine the stress tensor at this point. Take $\in = 210\times10^6$ kN/m^2, Poisson's ratio = 0.28. Also find Lame's constant.

6 a. Obtain the expression for compatibility equation in terms of stresses for plane strain problems in the absence of body force. 12

b. Show that $\left(Ae^{\alpha y} + Be^{-\alpha y} + Cye^{\alpha y} + Dye^{-\alpha y}\right)\sin \alpha x$ represents a stress function. 8

UNIT - IV

7. A rectangular beam section of depth 2h and is subjected to pure bending. Assuming the stress function as a polynomial of 3rd degree and using St. Venant's principle, obtain the solution to the problem. Derive an expression for displacement components. 20

8 a. Derive the expressions for radial and tangential stresses in the case of solid rotating disc and state their maximum values. 10

b. A thick cylinder of inner radius 10 cm and outer radius 15 cm is subjected to an internal pressure of 12 MPa. Determine the radial and Hoop stresses in cylinder at inner and outer surface. 10

UNIT - V

9 a. Obtain the strain displacement relationship for a CST element. 16

b. Write the shape function for a CST element. Sketch neatly the variation of shape function. 4

10. Evaluate the element stiffness matrix for the four noded quadrilateral element shown in Fig. 10(b), using 1st Gaussian quadrature $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$

Thickness = 20 mm, $E = 2\times10^3$ kN/mm^2 and $\mu = 0$. 20

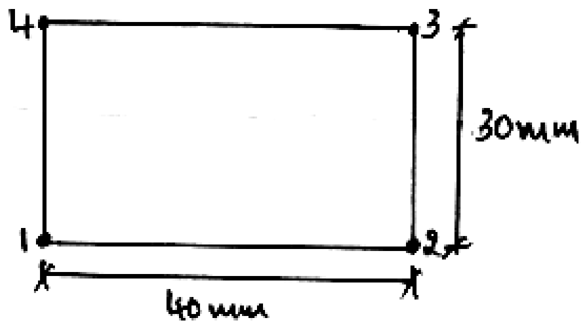

fig 10 (b).

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