



# P.E.S. College of Engineering, Mandya - 571 401

(An Autonomous Institution affiliated to VTU, Belagavi)

Fourth Semester, B.E. - Semester End Examination; July / Aug. - 2022

Complex Analysis, Statistics Probability and Numerical Techniques

(Common to all Branches)

Time: 3 hrs

Max. Marks: 100

## Course Outcomes

The Students will be able to:

CO1- Solve algebraic, transcendental and ordinary differential equations arising in various engineering flow and design data problems, using numerical techniques along with physical interpretation of the solutions associated with initial/boundary conditions.

CO2-Learn logical thinking and analytical / geometrical skills in linear algebra through vector spaces, basis, dimension and linear transformations along with construction a matrix of linear transformations with respect change of Bases of same or different dimensions. Understand iterative methods in linear algebra such as Gauss-Jacobi, Gauss-Seidel, Relaxation and Power method and their practical utility in engineering fields.

CO3-Understand the basics of functions of complex variables, analytic functions, conformal and bilinear transformations, complex integration, line / surface / volume integrals and residue theorems with their scientific / engineering importance.

CO4-Apply the basic tools of statistics to understand curve fitting, moments, skewness, kurtosis, correlation and regression, for frequency distributions; explore the idea of probability, probability distributions, required in the analysis of engineering experiments.

CO5-Apply the basic concepts of probability distributions to understand concept of joint probability and to find expectation covariance, correlation coefficient etc. and to understand probability vector, stochastic matrix etc. Obtain series solution of essential ODE's such as Bessel's and Legendre's differential equations and understand their scientific/engineering utility.

**Note:** I) PART - A is compulsory. Two marks for each question.

II) PART - B: Answer any **Two** sub questions (from a, b, c) for a Maximum of **18 marks** from each unit.

Q. No.	Questions	Marks	BLs	COs	POs
<b>PART - A</b>		<b>10</b>			
I a.	Write the formula of Runge-Kutta method of 4 <sup>th</sup> order.	2	L1	CO1	PO1
b.	Write the Rodrigue's formula of the legendre polynomials $P_n(x)$ .	2	L2	CO2	PO2
c.	Write Cauchy-Reimann equations in polar form.	2	L3	CO3	PO1
d.	Write the normal equations for fitting the straight line $y = a + bx$ .	2	L2	CO4	PO1
e.	Write the formula for mean and variance of binomial distribution.	2	L1	CO4	PO1
<b>PART - B</b>		<b>90</b>			
<b>UNIT - I</b>		<b>18</b>			
1 a.	By using the bisection method, find an approximate root of the equation, $\sin x = \frac{1}{x}$ , that lies between $x = 1$ and $x = 1.5$ (measured in radians). Carry out computations upto the 4 <sup>th</sup> stage.	9	L1	CO1	PO1
b.	Write Newton-Raphson iterative formula for $x_{n+1}$ . Use the Regula Falsi method to find the fourth root of 12 correct to three decimal places.	9	L2	CO1	PO2
c.	Write Milne's predictor-corrector formula. Using Euler's modified formulae, solve $\frac{dy}{dx} = x + y$ at $x = 0.2$ given that $y(0) = 1$ .	9	L3	CO1	PO2

**UNIT - II**

**18**

2 a. Employ Gauss-Seidel iteration method to solve,

$$5x + 2y + z = 12$$

$$x + 4y + 2z = 15$$

$$x + 2y + 5z = 20$$

9 L1 CO2 PO2

Carryout four iterations taking the initial approximation to the solution as (1, 0, 3).

b. Solve by relaxation method,

$$10x + 2y + z = 9$$

$$x + 10y - z = -22$$

$$-2x + 3y + 10z = 22$$

9 L2 CO2 PO2

c. Prove that:  $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \sin x$  and  $J_{-\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \cos x$ .

9 L1 CO2 PO2

**UNIT - III**

**18**

3 a. Show that  $f(z) = \cos z$  is analytic and hence find its derivatives.

9 L2 CO3 PO1

b. i) Show that:  $u = x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$  is harmonic.

ii) Given  $\phi(x, y) = \log(x^2 + y^2)$ , find the stream function  $\psi(x, y)$  and also find the complex potential.

9 L2 CO3 PO2

c. Discuss the conformal transformation  $w = e^z$ .

9 L2 CO3 PO1

**UNIT - IV**

**18**

4 a. Evaluate  $\int_C \frac{e^{2z}}{(z+1)(z-2)} dz$ , where  $C$  is the circle  $|z|=3$ .

9 L3 CO4 PO2

b. Calculate the co-efficient of correlation and also find lines of regression for the following data,

Student	A	B	C	D	E	F	G
$x$	1	2	3	4	5	6	7
$y$	9	8	10	12	11	13	14

9 L2 CO4 PO2

c. Define skewness and kurtosis.

The following table gives the results of the measurements of train resistance  $V$  velocity in miles per hour.  $R$  is the resistance in *lbs* (pounds) per ton.

$V$	-2	-1	0	1	2
$R$	-3.15	-1.39	0.62	2.88	5.378

9 L3 CO4 PO2

If  $R$  is related to  $V$  by the relation,  $R = av^2 + bv + c$ , find  $a, b, c$ .

UNIT - V

5 a. Find the constant  $K$  such that,

$$f(x) = \begin{cases} Kx^2 & 0 < x < 3 \\ 0 & \text{otherwise} \end{cases} \text{ is a p.d.f.}$$

Also compute;

i)  $p(1 < x < 2)$

ii)  $p(x \leq 1)$

iii) Mean

9 L3 CO4 PO1

b. Write the formula of mean and standard deviation in Binomial distribution.

In a certain factory turning out razor blades there is a small probability of  $\frac{1}{500}$

for any blade to be defective. The blades are supplied in packets of 10. Use Poisson distribution to calculate the approximate number of packets containing;

9 L2 CO4 PO2

i) No defective

ii) One defective

iii) Two defective blades in a consignment of 10,000 packets

c. D) The joint probability distribution of two random variables  $X$  and  $Y$  is given below.

$X \backslash Y$	-3	2	4
1	0.1	0.2	0.2
3	0.3	0.1	0.1

9 L3 CO4 PO2

Find; i) Marginal distributions of  $X$  and  $Y$

ii)  $E(X)$ ,  $E(Y)$  and  $E(XY)$

II) Find the unique probability vector for the regular stochastic matrix,

$$\begin{bmatrix} 0 & 1 \\ 1/2 & 1/2 \end{bmatrix}$$

\* \* \* \*