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P.E.S. College of Engineering, Mandya - 571 401

(An Autonomous Institution affiliated to VTU, Belagavi)

Fourth Semester, B.E. - Make-up Examination; March/April - 2022

Engineering Mathematics - IV

(Common to EC, EE, IS & CS Branches)

Time: 3 hrs *Max. Marks: 100*

Note: *Answer **FIVE** full questions, selecting **ONE** full question from each unit.*

UNIT - I

1 a. Use Newton-Raphson iterative formula to find the real root of the equation $x\log_{10} x = 1.2$ and hence find the root correct to three decimal places. 6

b. Find real root of the equation of $cosx = 3x - 1$ correct to three decimal places using Regula-falsi method 7

c. Find the smallest root of the equation $x^2 + 2x - 2 = 0$, using fixed point iteration method and accelerate the convergence by Aitkin's $\Delta^2 -$method. 7

2 a. Using modified Euler's method find y at $x = 0.2$ given, $\dfrac{dy}{dx} = 3x + \dfrac{1}{2}y$ with $y(0) = 1$ taking $h = 0.1$. Perform three iterations at each step. 6

b. Use Fourth order Runge-Kutta method to solve, $(x + y)\dfrac{dy}{dx} = 1,\ y(0.4) = 1$ at $x = 0.5$ correct to four decimal places. 7

c. Apply Milnes and Adam- Bash forth predictor &corrector method to compute $y(1.4)$ correct to four Decimal place given $\dfrac{dy}{dx} = x^2 + \dfrac{y}{2}$ and the data: $y(1) = 2,\ y(1.1) = 2.2156,\ y(1.2) = 2.4649,\ y(1.3) = 2.7514$ 7

UNIT - II

3 a. Define vector space and subspace. Give with suitable example. 6

b. Prove that $u = (1,0,0), v = (0,1,0)\, and\ w = (0,0,1)$ are linearly independent. 7

c. Find rank and nullity of the linear transformation $.T: R^3 \to R^3 by\ T(x,y,z) = (x + y,\ x + y + 2z, 2x + y + 3z)$ 7

4 a. Solve the system of the equations $x + y + 54z = 110,\quad 27x + 6y - z = 85,\quad 6x + 15y + 2z = 72$ by Gauss –Seidel method to obtain the numerical solution correct to three places of decimals. 6

b. Solve by relaxation method; $10x - 2y - 2z = 6\ ,\quad -x + 10y - 2z = 7,\quad -x - y + 10z = 8,$ 7

c. Find the dominant Eigen value and the corresponding Eigen vector of the matrix, by Power method $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ 7

Taking $[\,1\,,1\,.\,1\,]^{'}$ as initial Eigen vector (perform six iterations).

UNIT - III

5 a. Show that $f(z) = \sin z$ is analytic and hence find $f^{'}(z)$. 6

b. If $f(z)$ is analytic, show that $\left[\dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2}\right] |\, f(z)\,|^2 = 4\,|\, f^{'}(z)\,|^2$ 7

c. Discuss the conformal transformation $w = z + (1/z),\ z \neq 0$ 7

6 a. Evaluate $\int_0^{2+i} (\bar{z})^2 dz$ (i) along the line $x = 2y$ (ii) along the real axis up to 2 and then vertically to $2+i$. 6

b. Expand $f(z) = \dfrac{2z+3}{(z+1)(z-2)}$ as Laurent series in the regions 7

$(i)\ |Z| < 3 \qquad (ii)\ 1 < |Z| < 2$

c. Evaluate $\int \dfrac{\sin \pi z^2 + \cos \pi z^2}{(z-1)^2 (z-2)} dz$, where C is the circle $|Z| = 3$ .using Cauchy's residue theorem. 7

UNIT - IV

7 a. The first four moments about arbitrary value '4' of a frequency distribution are -1.5, 17, -30 and 108 respectively. Find skewness and kurtosis, based on moments. 6

b. Fit a parabola of second degree $y = a + bx + cx^2$ for the data 7

x	0	1	2	3	4
y	1	1.8	1.3	2.5	2.3

c. Find the equation of the lines of regression and correlation coefficient for the following data: 7

x	36	23	27	28	28	29	30	31	33	35
y	29	18	20	22	27	21	29	27	29	28

8 a. The p.m.f of a random variable X(= x) is given in the following table: 6

x	0	1	2	3	4	5	6
$f(x)$	k	$3k$	$5k$	$7k$	$9k$	$11k$	$13k$

For what value of k, this represents a valid probability distribution? Also, find $p(x \geq 5)$, $p(x \leq 4)$ and $p(3 < x \leq 6)$.

Contd...3

b. The number of telephone lines at an instant of time is a binomial variate with probability 0.1 that a line is busy. If 10 lines are chosen at random , what is the probability that (i) at least one line is busy (i)no line is busy (ii)all lines are busy(iii) at least one line is busy(iv) at most 2 lines are busy. 7

c. The length of a telephone conversation has an exponential distribution with mean of 3 minutes. Find the probability that a cell (i) ends in less than 3 minutes (ii) takes between 3 and 5 minutes. 7

UNIT - V

9 a. The joint distribution of two random variables X and Y is as follows. 6

X \ Y	-4	2	7
1	1/8	1/4	1/8
5	1/4	1/8	1/8

Compute the following:

i) $E(X) and E(Y)$ ii) $E(XY)$ iii) $\sigma_x and \sigma_y$

b. Find the unique fixed probability vector of the regular stochastic matrix 7

$$A=\begin{bmatrix} 0 & 1 & 0 \\ 1/6 & 1/2 & 1/3 \\ 0 & 2/3 & 1/3 \end{bmatrix}$$

c. The joint density function of two continuous random variables X and Y is given by $f(x,y)=\begin{cases} kxy, 0\le x\le 4, 0\le y\le 5 \\ o, otherwise \end{cases}$ Find; 7

i) The value of k ii) E(XY) iii) E(2X+3Y)

10 a. Obtain the series solution of the differential equation $\frac{d^2y}{dx^2}+xy=0$ 6

b. Express the polynomial in terms of Legendre polynomial x^3+x^2+x+1. 7

Prove that; i) $J_{1/2}(x)=\sqrt{\frac{2}{\pi x}}\sin x$ ii) $J_{-1/2}(x)=\sqrt{\frac{2}{\pi x}}\cos x$ 7

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