## U.S.N

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P.E.S. College of Engineering, Mandya - 571401
(An Autonomous Institution affiliated to VTU, Belagavi)
Third Semester, B.E. - Computer Science and Engineering
Semester End Examination; March / April - 2022
Discrete Mathematical Structures
Time: 3 hrs
Max. Marks: 100

## Course Outcomes

The Students will be able to:
CO1: Verify the correctness of an argument using propositional and predicate logic.
CO2: Demonstrate the ability to solve problems using counting techniques and Combinatorics in the context of discrete probability.
CO3: Solve problems involving recurrence relations.
CO4: Construct proofs using direct proof, proof by contraposition, proof by contradiction, and proof by cases, and mathematical induction.
CO5: Ability to Explain and distinguish graphs and their properties.
Note: I) PART - A is compulsory. Two marks for each question.
II) PART - B: Answer any Two sub questions (from $a, b, c$ ) for a Maximum of 18 marks from each unit.
Q. No.

## Questions

I : PART - A
Marks BLs COs
POs 10

I a. $p: A$ is a circle or a conic
$q: \sqrt{5}$ is a real number
r: Exponential series is convergent. Express the compound proportion $p \rightarrow(q \vee r)$ in words.
b. Obtain the recursive definition for the sequence $\left(a_{n}\right)$ given by, $a_{n}=n(n+2)$.
c. Let $f: R \rightarrow R$ defined by,

$$
f(x)=\left\{\begin{array}{cc}
3 x-5 & x>0 \\
-3 x+1 & x \leq 0
\end{array}\right.
$$

Determine: $f^{-1}(0), f^{-1}(1)$.
d. There are eight letters to eight different people to be placed in different addressed envelopes. Find the number of ways of doing this
$2 \quad \mathrm{~L} 2 \quad \mathrm{CO} 4 \quad \mathrm{PO} 1$ so that at least one letter gets to the right person.
e. Determine the number of different paths of length 2 in the graph shown below.


2 L2 CO 5 PO 2

## II : PART - B <br> UNIT - I

1 a. Prove that for any three propositions,

$9 \quad \mathrm{~L} 2 \quad \mathrm{CO} 1 \quad \mathrm{PO} 2$

9 L3 CO1 PO2
$\neg r \vee(\neg t \vee u)$
$\frac{p \wedge t}{\therefore u}$
c. Find whether the following argument is valid?

If a triangle has two equal sides then it is isosceles
If a triangle is isosceles. Then it has two equal angles
The triangle ABC does not have two equal angles.
$\therefore \mathrm{ABC}$ does not have two equal sides.
UNIT - II
18
2 a. If $n$ is any positive integer prove that,
$1.2+2.3+3.4+\cdots+n(n+1)=\frac{1}{3} n(n+1)(n+2)$ using mathematical induction.
b. A sequence $\left\{a_{n}\right\}$ is defined recursively by,
$a_{1}=4, a_{n}=a_{n-1}+n, n \geq 2$ find $a_{n}$ in explicit form.
c. I) How many arrangements are there for all letters in the word SOCIOLOGICAL?
II) In how many of these arrangements?
i) A and G are adjacent?
ii) All the vowels are adjacent?

UNIT - III
3 a. State Pigeon hole principle and extended Pigeon hole principle. A magnetic tape contains a collection of 5 lakh strings made up of four or fewer number of english letters. Can all the strings in the collection be distinct?
b. Let $A=\{1,2,3,4,5\}$. Define a relation $R$ on $A \mathrm{X} A$ by, $\left(x_{1}, y_{1}\right) R\left(x_{2}, y_{2}\right)$ If and only if $x_{1}+y_{1}=x_{2}+y_{2}$
i) Verify that $R$ is an equivalence relation on $A \times A$
ii) Determine the equivalence classes $[(1,3)],[(2,4)]$ and $[(1,1)]$
iii) Determine the partition of $A \times A$ induced by $R$.
c. Let $S=\{1,2,3\}$ and $P(S)$ be the power set of $S$ on $P(S)$. Define the relation $R$ by $X R Y$ if and only if $X \subseteq Y$. Show that this relation is a partial order on $P(S)$. Draw its Hasse diagram.

## UNIT - IV

18
4 a . Find the rook polynomial for the board shown below

| 1 | 2 |  |  |
| :--- | :--- | :--- | :--- |
| 3 | 4 |  | 5 |
|  | 6 | 7 | 8 |

b. Solve the recurrence relation $a_{n}-3 a_{n-1}=5 \times 3^{n}$ for $n \geq 1$ given that $a_{0}=2$.
c. How many integers between 1 and 300 (inclusive) are,
i) Divisible by at least one of $5,6,8$ ?
ii) Divisible by none of $5,6,8$ ?

UNIT - V
18
5 a. Show that the following two graphs are isomorphic


9 L2 CO5 PO1

9 L2 CO5 PO2
$9 \quad \mathrm{~L} 3 \quad \mathrm{CO} 5 \mathrm{PO} 2$
"LETTER RECEIVED". Indicate the code.

