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P.E.S. College of Engineering, Mandya - 571 401

(An Autonomous Institution affiliated to VTU, Belagavi)

Third Semester, B.E. - Information Science and Engineering

Semester End Examination; March / April - 2022

Discrete Mathematics and Applications

Time: 3 hrs *Max. Marks: 100*

Course Outcomes

The Students will be able to:

CO1: Verify the correctness of an argument using propositional and predicate logic.

CO2: Demonstrate the ability to solve problems using counting techniques and Combinatorics in the context of discrete probability.

CO3: Solve problems involving recurrence relations.

CO4: Construct proofs using direct proof, proof by contraposition, proof by contradiction, proof by cases, and mathematical induction.

CO5: Ability to Explain and distinguish graphs and their properties.

Note: ***I) PART - A*** *is compulsory.* ***Two*** *marks for each question.*

II) PART - B*: Answer any* ***Two*** *sub questions (from a, b, c) for Maximum of* ***18 marks*** *from each unit.*

Q. No.	Questions	Marks
	I : PART - A	**10**
I a.	Negate and simplify the following: $\forall x, \ \left[p(x) \rightarrow q(x) \right]$.	2
b.	In how many ways can we distribute 10 identical marbles among 6 distinct containers?	2
c.	Define partition of a set. Let $A = \{1, 2, 3, 4\}$, write any two partition of A.	2
d.	Find the number of derangements of 1, 2, 3, 4.	2
e.	Define Euler's circuit and Euler trials.	2
	II : PART - B	**90**
	UNIT - I	**18**
1 a.	Prove the following logical equivalence without using truth table: i) $(p \rightarrow q) \wedge \left[\neg q \wedge (r \vee \neg q) \right] \Leftrightarrow \neg (q \vee p)$ ii) $\left[\neg p \wedge (\neg q \wedge r) \right] \vee (q \wedge r) \vee (p \wedge r) \Leftrightarrow r$	9
b.	Establish the validity of the argument, i) $p \rightarrow q$ $q \rightarrow (r \wedge s)$ $\neg r \vee (\neg t \vee u)$ $p \wedge t$ $\therefore u$ ii) $(\neg p \vee \neg q) \rightarrow (r \wedge s)$ $r \rightarrow t$ $\neg t$ $\therefore p$	9

Contd... 2

c. i) Write the converse, inverse and contra positive of the implication, "If 0 + 0 = 0 then 1 + 1 = 1" 3

ii) Give indirect proof and proof by contradiction for the statement, "If n is odd integers then $n + 9$ is an even integer" 6

UNIT - II **18**

2 a. State well ordering principle. If n is any positive integers, prove that $1.2+2.3+3.4+........+n.(n+1)=\frac{1}{3}n(n+1)(n+2)$ using mathematical induction. 9

b. i) Find an explicit definition of the sequence whose recursive definition is, $a_n = 2a_{n-1}+1$ for $n \geq 2$, $a_1 = 7$ 4

ii) Find the number of permutations of the letters of the word MASSASAUGA. In how many of these, all four A's are together? 5

c. A women has 11 close relatives and she wishes to invite 5 of them to dinner. In how many ways can she invite them in the following situations:

i) There is no restriction on the choice 9

ii) Two particular persons will not attend separately

iii) Two particular persons will not attend together

UNIT - III **18**

3 a. Define equivalence relation. Let A = {1, 2, 3, 4, 5}, define a relation R on A x A by (x_1, y_1) R (x_2, y_2) iff $x_1 + y_1 = x_2 + y_2$. Show that R is an equivalence relation. Determine equivalence classes [(1, 3)] and [(2, 5)]. 9

b. i) In a set of how many people, at least 5 persons must have been born on same day of the week. 5

ii) Define the terms: 4

Invertible function, Onto function, Reflexive relation, Asymmetric relation

c. Consider the following Hasse Diagram shown in Fig 3c,

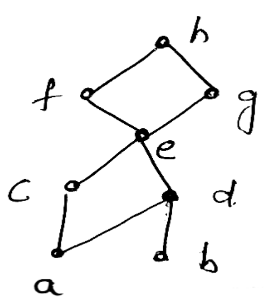
i) Find maximal and minimal elements

ii) Find greatest and least element

iii) If $B \subseteq A$ and $B = \{c, d, e\}$, find lower bounds of B and GLB of B

iv) If $B \subseteq A$ and $B = \{a, c\}$, find upper bounds of B and $L\cup B$ of B 9

Fig 3c.



UNIT - IV **18**

4 a. How many integers between 1 and 300 (inclusive) are divisible by at least one of 5, 6, 8? and divisible by none of 5, 6, 8. 9

b. Five teachers T_1, T_2, T_3, T_4 and T_5 are to be made class teachers for 5 classes C_1, C_2, C_3, C_4 and C_5. One teacher for each class. T_1 and T_2 do not wish to become the class teachers for C_1 or C_2. T_3 and T_4 for C_4 and T_5 for C_3 or C_4 or C_5. In how many ways can the teachers be assigned the work? 9

c. Solve the following recurrence relation:

$F_{n+2} = F_{n+1} + F_n$ for $n \geq 0$ 9

$F_0 = 0$ and $F_1 = 1$

UNIT - V **18**

5 a. Define bipartite graph. Let $G = (V, E)$ be a simple graph of order 'n' and size 'm'. If G is a bipartite graph, prove that $4m \leq n^2$. 9

b. With suitable example, discuss the following: 9

i) Isomorphism

ii) Planar graph

c. i) Prove that a tree with n vertices has $n-1$ edges 4

ii) Construct an optimal prefix code for the symbols a, o, q, u, y, z that occur with frequencies 20, 28, 4, 17, 12, 7 respectively 5

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