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P.E.S. College of Engineering, Mandya - 571 401

(An Autonomous Institution affiliated to VTU, Belagavi)

Third Semester, B.E. - Semester End Examination; March / April - 2022

Transform Calculus, Fourier Series and Numerical Techniques

Time: 3 hrs *Max. Marks: 100*

Course Outcomes

The Students will be able to:

CO1: Apply forward, backward difference formulae and central differences formulae in solving interpolation-extrapolation problems in engineering field.

CO2: Numerical differentiation and integration rules in solving engineering where the handlings of numerical methods are inevitable.

CO3: Apply the knowledge of periodic function, Fourier series, complex Fourier series, Fourier sine/cosine series of a function valid in different periods. Analyze engineering problems arising in control theory/fluid flow phenomena using harmonic analysis.

CO4: Understand complex/infinite Fourier transforms Fourier sine and Fourier cosine transforms with related properties. Analyze the engineering problems arising in signals and systems, digital signal processing using Fourier transform techniques. Define Z-transforms& find Z-transforms of standard functions to solve the specific problems by using properties of Z-transforms. Identify and solve difference equations arising in engineering applications using inverse Z-transforms techniques.

CO5: Define Partial Differential Equations (PDE's), order, degree and formation of PDE's and, to solve PDE's by various methods of solution. Explain one - dimensional wave and heat equation and Laplace's equation and physical significance of their solutions to the problems selected from engineering field.

Note: ***I) PART - A*** *is compulsory.* ***Two*** *marks for each question.*

II) PART - B*: Answer any* ***Two*** *sub questions (from a, b, c) for Maximum of* ***18 marks*** *from each unit.*

Q. No.	Questions	Marks	BLs	COs	POs
	I : PART - A	**10**			
I a.	Write the Bessel's formula upto the fourth order differences.	2	L1	CO1	PO1
b.	The first order derivative using forward interpolation formula at the point x_0 upto fourth order is equal to?	2	L1	CO2	PO1
c.	If $f(x)$ is an odd function over the interval $(-l, l)$, then write the Euler's formulae for $f(x)$.	2	L1	CO3	PO1
d.	The Z-transform of a^n and e^{an} is equal to?	2	L1	CO4	PO1
e.	Verify that $z=\left(x^2+a\right)\left(y^2+b\right)$ is a complete solution of the first order PDE $pq=4xyz$.	2	L2	CO5	PO2
	II : PART - B	**90**			
	UNIT - I	**18**			
1 a.	Define extrapolation. From the following table, find the number of students who obtained marks between 40 and 45.	9	L2	CO1	PO2

Marks	30 - 40	40 - 50	50 - 60	60 - 70	70 - 80
No. of Students	31	42	51	35	31

Q. No.	Questions	Marks	BLs	COs	POs
b.	Certain corresponding values of x and $\log_{10}(x)$ are (300, 2.4771), (304, 2.4829), (305, 2.4843) and (307, 2.4871). Find; i) $log_{10}(301)$ and ii) $log_{10}(306)$, by using Lagrange interpolation formula.	9	L2	CO1	PO1

c. Use Gauss's forward formula to evaluate y_{30}, given that $y_{21} = 18.4708$, $y_{25} = 17.8144$, $y_{29} = 17.1070$, $y_{33} = 16.3432$ and $y_{37} = 15.5154$. 9 L2 CO1 PO1

UNIT - II

18

2 a. Find $f'(10)$ from the following data: 9 L2 CO2 PO1

x	3	5	11	27	34
$f(x)$	−13	23	899	17315	35606

b. Evaluate $\int_0^6 \frac{dx}{1+x^2}$, by using, i) Trapezoidal rule and ii) Simpson's $\left(\frac{3}{8}\right)^{th}$ rule. 9 L2 CO2 PO2

c. Evaluate: $\int_4^{5.2} \log_e x\, dx$ using Simpson's $\left(\frac{1}{3}\right)^{rd}$ rule and Weddle's rule, taking 7 ordinates. 9 L2 CO2 PO2

UNIT - III

18

3 a. Find the Fourier series for the function $f(x) = |x|$ in $-\pi \le x \le \pi$. Hence deduce that $\frac{\pi^2}{8} = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^2}$. 9 L2 CO3 PO2

b. Obtain the Fourier series for the function $f(x) = \begin{cases} \pi x & \text{in } 0 \le x \le 1 \\ \pi(2-x) & \text{in } 1 \le x \le 2 \end{cases}$ 9 L2 CO3 PO2

c. A function $f(x)$ of period 2π is specified by the following table. 9 L3 CO3 PO2

x	0	$\frac{\pi}{3}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{5\pi}{3}$	2π
$f(x)$	7.9	7.2	3.6	0.5	0.9	6.8	7.9

Obtain the Fourier series of $f(x)$ upto the first harmonic.

UNIT - IV

18

4 a. Find the Fourier transform of $f(x) = \begin{cases} 1-|x|, & \text{for } |x| \le 1 \\ 0, & \text{for } |x| > 1 \end{cases}$. Hence evaluate $\int_0^{\infty} \frac{\sin^2 t}{t^2}\, dt$. 9 L3 CO4 PO2

b. Find the Fourier sine and cosine transform of e^{-ax} where $a > 0$. 9 L2 CO4 PO1

c. Obtain the Z-transform of $\cosh n\theta$ and $\sinh n\theta$. 9 L2 CO4 PO1

UNIT - V

18

5 a. i) Form a partial differential equation by eliminating arbitrary function from $z = y^2 + 2f\left(\frac{1}{x} + \log y\right)$. 4

ii) Find the general solution of the equation $yz\frac{\partial z}{\partial x} + zx\frac{\partial z}{\partial y} = xy$. 5

L2 CO5 PO1

b. Use the method of separation of variables to solve $\frac{\partial z}{\partial x} = 2\frac{\partial z}{\partial y} + z$, given $z(x,0) = 6e^{-3x}$. 9 L3 CO5 PO2

c. Find the various possible solutions of the two dimensional Laplace's equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$. 9 L2 CO5 PO2

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