U.S.N

P.E.S. College of Engineering, Mandya - 571 401

(An Autonomous Institution affiliated to VTU, Belagavi)

Fourth Semester, B.E. - Semester End Examination; July / August - 2022

Engineering Mathematics - IV

(Common to EE, EC, CS&E, IS&E Branches)

Time: 3 hrs *Max. Marks: 100*

***Note**: Answer **FIVE** full questions, selecting **ONE** full question from each unit.*

Q. No.	Questions	
	UNIT - I	
1 a.	Find a real root of the equation $x\log_{10} x = 1.2$ convert to five decimal places using the Newton-Raphson method.	6
b.	Find real root of an equation $\cos x = 3x - 1$ correct to three decimal place using Regula falsi method.	7
c.	Find a real root of the equation $x^3 - x - 1 = 0$ using fixed point iteration method. Acceleration the convergence by Aitkin's Δ^2 method carry out three iterations.	7
2 a.	Using Runge-Kutta method of fourth order. Find y (0.2) for the equation $\frac{dy}{dx} = \frac{y-x}{y+x}$, y(0) = 1 taking h = 0.1	6
b.	Apply modified Euler's method to find $y(0.1)$ given that $\frac{dy}{dx} = x^2 + y, \quad y(0) = 1$, by taking h = 0.05. Considering two approximations in each step.	7
c.	Apply Adam-Bashforth method to solve the equation $(y^2+1)dy - x^2 dx = 0$ *at* $x = 1$ given data $\mathrm{y}(0) = 1$, $\mathrm{y}(0.25) = 1.0026$, $\mathrm{y}(0.5) = 1.0206$ and $\mathrm{y}(0.75) = 1.0679$ (applying corrector formula twice).	
	UNIT - II	
3 a.	Define vector space and subspace. Give with suitable examples	6
b.	Find the change of basis matrix P from usual basis, $\mathrm{E} = \{\mathrm{e}_1, \mathrm{e}_2, \mathrm{e}_3\}$ of R^3 to the basis $\mathrm{S} = \{\mathrm{w}_1 = (1,1,1), \mathrm{w}_2 = (1,1,0), \mathrm{w}_3 = (1,0,0)\}$	7
c.	Find Rank and nullity of the linear transformation. $\mathrm{T} = \mathrm{R}^3 \to \mathrm{R}^3$ by $\mathrm{T(x,y,z) = (x+z, x+y+2z, 2x+y+3z)}$	5
4 a.	Apply Gauss-Seidal iterative method to solve the equations $27x + 6y - z = 85$, $6x + 15y + 2z = 72$, $x + y + 54z = 110$ perform three iterations.	6
b.	Solve by relaxation method: $10x - 2y - 2z = 6$, $-x + 10y - 2z = 7$, $-x - y + 10z = 8$	7

c. Determine the largest Eigen value and the corresponding Eigen vector of the matrix

$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$ taking initially $[1, 0, 0]^T$ (perform six iterations) 7

UNIT – III

5 a. Show that $f(z) = \sin z$ is analytic and hence find $f^1(z)$. 6

b. Find analytic function f(z) as a function of z given that the sum of its real and imaginary part is $x^3 - y^3 + 3xy\,(x-y)$. 7

c. Find the bilinear transformation that maps the points $Z_1 = 0$, $Z_2 = -i$, $Z_3 = 2i$ into $W_1 = 5i$, $W_2 = \infty$, $W_3 = -i/3$ 7

6 a. Evaluate $\int_0^{2+i} (\bar{z})^2\, dz$ along the line x = 2y 6

b. Expand $f(z) = \dfrac{1}{(z-1)(2-z)}$ as Laurent's series valid, for 7

i) $|z| < 1$ ii) $1 < |z| < 2$.

c. Using the Cauchy's residue theorem 7

$\int_c \dfrac{\sin \pi z^2 + \cos \pi z^2}{(z-1)^2(z-2)}$ where C is the circle $|z| = 3$

UNIT - IV

7 a. In a certain distribution $\{x_i\ f_i\}\ i = 1, 2, \ldots n$. The first four moment about the point '5' are – 1.5, 17, - 30 and 108. Calculate Skewness and kurtosis. 6

b. Fit a parabola for the following data: 7

X:	1	2	3	4	5	6	7	8	9
Y:	2	6	7	8	10	11	11	10	9

c. If θ is the angle between the two regression lines, show that $\tan\theta = \left(\dfrac{1-r^2}{r}\right)\dfrac{\sigma x \sigma y}{\sigma^2 x + \sigma^2 y}$ 7

8 a. The probability distribution of a finite random variable x (x) is given by the following table find the value of K, mean and variance 6

X_i :	-2	-1	0	1	2	3
$P(x_i)$:	0.1	k	0.2	2k	0.3	k

b. The probability that a pen manufactured by a factory be defective is, $\frac{1}{10}$. If 12 such pens are manufactured. What is the probability that 7

i) Exactly 2 are defective

ii) At least 2 are detective

iii) None of them are detective

c. In a test an electric bulbs, it has found that the life time of a particular brand was distributed normally with an average life of 2000 hours and S.D of 60 hours. In a purchase 2500 bulbs, find the number of bulbs that are likely to last for

i) More than 2100 hours 7

ii) Less than 1950 hours

iii)Between 1900 to 2100 hours

Given ϕ (1.67) = 0.4525 ϕ (0.83) = 0.2967

UNIT - V

9 a. The joint distribution of two random variables x and y is as follows: 6

X \ Y	−4	2	7
1	1/8	¼	1/8
5	¼	1/8	1/8

Find; i) E(x) and E(y) ii) E(xy) iii) σ_x and σ_y

b. Find the unique fixed probability vector for the regular stochastic matrix; 7

$$A = \begin{bmatrix} 0 & 1 & 0 \\ \frac{1}{6} & \frac{1}{2} & \frac{1}{3} \\ 0 & \frac{2}{3} & \frac{1}{3} \end{bmatrix}$$

c. If x and y are continuous random variables the joint density function 7

$$f(x, y) = \begin{cases} c(x^2 + y^2) : 0 \le x \le 1 \\ 0, \textit{ other wise} \end{cases}$$

Determine; i) Constant C ii) P (x < ½, y > ½) iii) P (¼ < x < ¾)

10 a. Obtain the series solution of the differential equation $\frac{d^2 y}{dx^2} + xy = 0$ 6

b. Obtain $J_n(x)$ as a solution of the Bessel's differential equation $x^2 y'' + xy' + (x^2 - n^2) y = 0$ 7

c. State Rodrigue's formula express $x^3 + x^2 + x + 1$ in terms of Legendre's polynomials. 7

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