



# P.E.S. College of Engineering, Mandya - 571 401

(An Autonomous Institution affiliated to VTU, Belagavi)

Fourth Semester, B.E. - Semester End Examination; Sep. / Oct. - 2023

## Applied Mathematical Methods

(Common to AU, CV, IP and ME)

Time: 3 hrs

Max. Marks: 100

### Course Outcomes

The Students will be able to:

CO1: Apply the concepts of an analytic function and their properties to solve the problems arising in engineering field

CO2: Use the concept of correlation and regression analysis to fit a suitable mathematical model for the statistical samples arise in engineering field

CO3: Explain various numerical techniques to solve equations approximately having no analytical solutions.

CO4: Interpret discrete and continuous probability distributions in analyzing the probability models and solve problems involving Markov chains

CO5: Estimate the series solutions of ordinary difference equation.

**Note:** I) PART - A is compulsory. Two marks for each question.

II) PART - B: Answer any **Two** sub questions (from a, b, c) for a Maximum of **18** marks from each unit.

Q. No.	Questions	Marks	BLs	COs	POs												
<b>I : PART - A</b>		<b>10</b>															
1 a.	Define analytic function.	2	L1	CO1	PO1												
b.	Write Karl Pearson's coefficient of correlation formula.	2	L1	CO2	PO1												
c.	Write Runge Kutta method of fourth order formula.	2	L1	CO3	PO1												
d.	State Binomial distribution.	2	L1	CO4	PO1												
e.	Write the Bessel's differential equation.	2	L1	CO4	PO1												
<b>II : PART - B</b>		<b>90</b>															
<b>UNIT - I</b>		<b>18</b>															
2 a.	Show the $u = e^x (x \cos y - y \sin y)$ is harmonic and find its harmonic conjugate. Also determine the corresponding analytic function.	9	L2	CO1	PO2												
b.	Show the $w = z + e^z$ is analytic and hence find $\frac{dw}{dz}$ .	9	L2	CO1	PO1												
c.	Define conformal transformation. Find the bilinear transformation which map the points $Z = 1, i, -1$ into $w = i, 0, -i$ .	9															
<b>UNIT - II</b>		<b>18</b>															
3 a.	Expand $f(z) = \frac{1}{(z-1)(2-z)}$ as a Laurent series valid for;	9	L2	CO2	PO1												
	i) $ Z  < 1$ ii) $1 <  Z  < 2$																
b.	Fit parabola $y = ax^2 + bx + c$ by the method of least squares for the data																
	<table border="1" style="margin-left: 40px;"> <tr> <td>x</td> <td>0</td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> </tr> <tr> <td>y</td> <td>1</td> <td>1.8</td> <td>1.3</td> <td>2.5</td> <td>2.3</td> </tr> </table>	x	0	1	2	3	4	y	1	1.8	1.3	2.5	2.3	9	L2	CO2	PO2
x	0	1	2	3	4												
y	1	1.8	1.3	2.5	2.3												

- c. Calculate the coefficient of correlation and obtain the lines of regression for the following data:

$x$	1	2	3	4	5	6	7	8	9
$y$	9	8	10	12	11	13	14	16	15

9 L1 CO2 PO2

Obtain an estimate for  $y$  which corresponds to  $x = 6.2$

**UNIT - III**

**18**

- 4 a. Write Newton-Raphson's iterative formula for  $X_{n+1}$ .

Using Regular-Falsi method find the approximate root of the equation  $xe^x = \cos x$  that lies between 0.4 and 0.6 ( $x$  is in radians) correct to 4 decimal places.

9 L2 CO3 PO2

- b. Given,  $\frac{dy}{dx} = 3x + \frac{y}{2}$ ,  $y(0) = 1$ . Compute  $y(0.2)$  by taking  $h = 0.2$  using Runge Kutta method of fourth order.

9 L2 CO3 PO1

- c. Employ Gauss-Seidel iteration method to solve  $5x + 2y + z = 12$ ,  $x + 4y + 2z = 15$ ,  $x + 2y + 5z = 20$  carryout 4 iterations taking the initial approximation to the solution as (1, 0, 3).

9 L1 CO3 PO1

**UNIT - IV**

**18**

- 5 a. The probability distribution of a finite random variable  $x$  is given by the following table:

$x_i$	-2	-1	0	1	2	3
$P(x_i)$	0.1	$K$	0.2	$2k$	0.3	$K$

9 L2 CO4 PO2

i) Find the value of  $K$  and calculate the mean and variance

ii) Find  $P(x < 1)$ ,  $P(-1 < x \leq 2)$   $P(x > -1)$

- b. Write the formula of mean and standard deviation in Poison's distribution. The probability that a man aged 60 will live to be 70 is 0.65. What is the probability that out of 10 men, now aged 60, at least 7 will live to be 70?

9 L3 CO4 PO2

- c. A coin is tossed three times. Let  $x$  denote 0 or 1 according as a tail or a head occurs on the first toss. Let  $y$  denote the total number of tails which occur. Determine;

9 L2 CO4 PO1

i) The joint distribution of  $x$  and  $y$

ii) Marginal distribution of  $x$  and  $y$

iii)  $E(x+y)$  and  $E(xy)$

**UNIT - V**

**18**

- 6 a. Find the power series solution of  $(1-x^2)y'' - 2xy' + 2y = 0$  about  $x = 0$ .

9 L2 CO4 PO1

- b. Solve the Bessel's differential equation  $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + (x^2 - n^2)y = 0$ .

9 L3 CO4 PO2

- c. Express  $x^3 + 2x^2 - 4x + 5$  in terms of Legendre's polynomials.

9 L3 CO4 PO2