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P.E.S. College of Engineering, Mandya - 571 401

(An Autonomous Institution affiliated to VTU, Belagavi)

Fourth Semester, B.E. - Semester End Examination; Sep. / Oct. - 2023

Applied Mathematical Methods

(Common to CS, EC, EE and IS)

Time: 3 hrs *Max. Marks: 100*

Course Outcomes

The Students will be able to:

CO1: Apply the concepts of an analytic function and their properties to solve the problems arising in engineering field.

CO2: Use the concept of correlation and regression analysis to fit a suitable mathematical model for the statistical samples arise in engineering field.

CO3: Apply the acquired knowledge of numerical technique to solve equations approximately having no analytical solutions.

CO4: Explain discrete and continuous probability distributions in analyzing the probability models and solve problems involving Markov chains.

Note: ***I) PART - A*** *is compulsory.* ***Two*** *marks for each question.*

II) PART - B*: Answer any* ***Two*** *sub questions (from a, b, c) for a Maximum of* ***18 marks*** *from each unit.*

Q. No.	Questions	Marks	BLs	COs	POs
	I : PART - A	**10**			
1 a.	Define bilinear transformation of $w = f(z)$.	2	L1	CO1	PO1
b.	Write normal equations to fit $y = ax + b$.	2	L1	CO2	PO1
c.	Define Eigen value and Eigen vector.	2	L1	CO3	PO1
d.	Define Random Variable.	2	L1	CO4	PO1
e.	Define; i) Probability vector and ii) Stochastic matrix.	2	L1	CO4	PO1
	II : PART - B	**90**			
	UNIT - I	**18**			
2 a.	i) Define Analytic function. ii) Show that $w = z + e^z$ is analytic and hence find $\frac{dw}{dz}$.	9	L1	CO1	PO1
b.	Show that $u = e^x (x\cos y - y\sin y)$ is harmonic and find its harmonic conjugate. Also determine the corresponding analytic function.	9	L3	CO1	PO2
c.	i) Define conformal transformation. ii) Find the bilinear transformation which maps the points $z = (1, i, -1)$ into $w = (i, 0, -i)$.	9	L3	CO1	PO2
	UNIT - II	**18**			
3 a.	State Cauchy's integral theorem and hence evaluate $\int_C \frac{e^{2z}}{(z+1)(z-2)}dz$, where C is the circle $\lvert z\rvert = 3$.	9	L2	CO2	PO2
b.	Fit a parabola $y = ax^2 + bx + c$ by the method of least squares for the data.	9	L2	CO2	PO1

x :	1	2	3	4	5	6	7	8	9
y :	2	6	7	8	10	11	11	10	9

c. Compute the coefficient of correlation and hence the equation of the lines of regression for the data. 9 L2 CO2 PO1

x :	1	2	3	4	5	6	7
y:	9	8	10	12	11	13	14

UNIT - III **18**

4 a. i) Write the false position formula for the root of $f(x) = 0$, in (a, b).

ii) Use Newton-Raphson method to derive an iterative formula to find $\sqrt{N}$ and hence find $\sqrt{12}$ correct to four decimal places. 9 L2 CO3 PO1

b. Using modified Euler's method, find y at $x = 1.2$ and 1.4 given,
$\frac{dy}{dx} = 1 + \frac{y}{x}$, $y_0 = 2$ at $x_0 = 1$, $h = 0.2$ 9 L3 CO3 PO2

c. Solve the system of equations:

$5x + 2y + z = 12$

$x + 4y + 2z = 15$

$x + 2y + 5z = 20$

By Gauss-Seidel method correct to three decimal places.

Take; $x^{(0)} = [0, 0, 0]$. 9 L2 CO3 PO2

UNIT - IV **18**

5 a. A random variable x has the following probability function for various values of x. Find K, $P(x < 6)$, $P(x \geq 6)$, $P(3 < x \leq 6)$ 9 L2 CO4 PO2

x:	0	1	2	3	4	5	6	7
$P(x)$:	0	K	$2K$	$2K$	$3K$	K^2	$2K^2$	$7K^2+K$

b. i) Write the formula for mean and standard deviation of binomial distribution.

ii) A shop has 4 diesel generator sets which it hires every day. The demand for a generator set on an average is a Poisson variate with value 5/2. Obtain the probability that on a particular day a demand had to be refused. 9 L3 CO4 PO2

c. The joint probability distribution of two random variables X and Y is as follows: 9 L2 CO4 PO1

X \ Y	−2	−1	4	5
1	0.1	0.2	0	0.3
2	0.2	0.1	0.1	0

Compute; i) $E(X)$, $E(Y)$ ii) $E(XY)$

iii) $\sigma x, \sigma y$ iv) Covariance of X and Y

Contd….3

UNIT - V **18**

6 a. i) Define regular stochastic matrix.

ii) Show that the Markov chain whose transition probability matrix

$$P = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$$

Is irreducible. Find the corresponding stationary probability vector. 9 L1 CO4 PO2

b. A population consists of 4 numbers 3, 7, 11, 15 consider all possible samples of size 2 which can be drawn from this population.

i) Find the mean and standard deviation of the population

ii) Find the mean and standard deviation of the sampling distribution of means 9 L2 CO4 PO2

iii) Considering samples without replacement find the mean and standard deviation of the sampling distribution of means

c. Five dice were thrown 96 times and the numbers 1, 2, of 3 appearing on the face of the dice follows the frequency distribution as below:

No. of dice showing, 1, 2, or 3	5	4	3	2	1	0
Frequency	7	19	35	24	8	3

Test the hypothesis that the data follows a binomial distribution: 9 L3 CO4 PO2

$\left(x^2_{0.05} = 11.07 \text{ for } 5 \text{ d.f}\right)$

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