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P.E.S. College of Engineering, Mandya - 571 401

(An Autonomous Institution affiliated to VTU, Belagavi)

Second Semester, B.E. - Semester End Examination; Sep. / Oct. - 2023

Integral Calculus, Partial Differential Equations and Numerical Methods

(Common to CS, IS, and AI&ML)

Time: 3 hrs *Max. Marks: 100*

Course Outcomes

The Students will be able to:

*CO1: **Knowledge** to Evaluate double and triple integration and identify the scalar, vector notation of functions of two and three dimensions ,recognize the partial differential equations and Numerical differences.*

*CO2: **Understand** to explain Area, Volume by double integration, change to polar coordinates describe divergence and flux in vector field; classify method of solutions of PDE's, Numerical differentiation and integrations.*

*CO3: **Apply** the Mathematical properties to evaluate triple integral and improper integral to interpret the irrotational and solenoidal vector field, find the solutions to problem arises in engineering field*

*CO4: **Analyze** multiple integrals ,vector differentiations and integration, the Mathematical model by partial differential equations, Numerical solution to algebraic and transcendental, ordinary differential equations and familiarize with modern mathematical tools namely SCILAB/PYTHON/MATLAB*

***Note:** **I) PART - A** is compulsory. **Two** marks for each question.*

***II) PART - B**: Answer any **Two** sub questions (from a, b, c) for a Maximum of **18 marks** from each unit.*

Q. No.	Questions	Marks	BLs	COs	POs
	I : PART - A	**10**			
1 a.	Write the integral expression for volume of a solid obtained by the revolution of a curve enclosing an area A about the initial line in the polar form.	2	L1	CO1	PO1
b.	Define Solenoidal and Irrotational vector.	2	L1	CO2	PO1
c.	Write the order and degree of the partial differential equation: $\frac{\partial^3 z}{\partial^2 x \partial y} = \sin(3x + 2y)$	2	L1	CO3	PO1
d.	Write the Weddle's formula to evaluate $\int_a^b f(x)dx$ for $n = 6$.	2	L1	CO4	PO1
e.	Write Modified Euler's formula.	2	L1	CO4	PO1
	II : PART - B	**90**			
	UNIT - I	**18**			
2 a.	Evaluate $\iint_R xy\,dx\,dy$ where R is the region bounded by the x-axis, ordinate $x = 2a$ and the curve $x^2 = 4ay$.	9	L1	CO1	PO1
b.	Evaluate $\int_0^1 \int_x^1 \frac{x}{\sqrt{x^2 + y^2}}\,dy\,dx$ by changing the order of integration.	9	L2	CO1	PO1
c.	i) Define Beta and Gamma function. ii) Show that $\int_0^{\pi/2} \sin^p\theta\,d\theta \times \int_0^{\pi/2} \sin^{p+1}\theta\,d\theta = \frac{\pi}{2(p+1)}$	9	L3	CO1	PO2
	UNIT - II				
3 a.	If $\varphi = x^2y^2z^3$ and $\vec{F} = 2x\,\hat{i} + 3y\,\hat{j} + 4z\,\hat{k}$ find the following: i) $\vec{F} \cdot \nabla(\varphi)$, ii) $\vec{F} \times \nabla(\varphi)$ iii) $\nabla(\lvert\vec{F}\rvert)$	9	L2	CO2	PO1

Q. No.	Question	Marks	Level	CO	PO
b.	Show that $\vec{F} - (2xy^2 + yz)\hat{i} + (2x^2y + xz + 2yz^2)\hat{j} + (2y^2z + xy)\hat{k}$ is a conservative force field. Find its scalar potential.	9	L2	CO2	PO1
c.	Verify Green's theorem for $\int_c (xy + y^2)dx + x^2\,dy$ where c is the closed curve of the region bounded by $y = x$ and $y = x^2$.	9	L2	CO2	PO2
	UNIT - III	**18**			
4 a.	i) Form the partial differential equation, by eliminating the arbitrary constants: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where a and b are arbitrary constants. ii) Find the partial differential equation arising from the equation: $\varphi(x + y + z, xy + z^2) = 0$, where φ is an arbitrary function.	9	L2	CO3	PO2
b.	Solve the partial differential equation: $\frac{\partial^3 z}{\partial x^3} + 4\frac{\partial z}{\partial x} = 0$, given that $z = 0$, $\frac{\partial z}{\partial x} = 0$, $\frac{\partial^2 z}{\partial x^2} = 4$, when $x = 0$.	9	L3	CO3	PO2
c.	Find the various possible solutions of the one dimensional wave equation $\frac{\partial^2 u}{\partial t^2} = c^2\frac{\partial^2 u}{\partial x^2}$ by the method of separation of variables.	9	L3	CO3	PO2
	UNIT - IV	**18**			
5 a.	From the following data estimate the number of students who have scored marks less than 70 by using suitable interpolation formula.	9	L2	CO4	PO2

Marks	0 - 20	20 - 40	40 - 60	60 - 80	80 - 100
Number of Students	41	62	65	50	17

Q. No.	Question	Marks	Level	CO	PO
b.	i) Write the Newton's divided difference formula for $y = f(x)$ up to third order ii) A function is specified by the following table: (see table below) Find the value of $f'(1.1)$.	9	L3	CO4	PO2

x	1	1.2	1.4	1.6	1.8	2.0
y	0.0	0.128	0.544	1.296	2.432	4.00

Q. No.	Question	Marks	Level	CO	PO
c.	Evaluate $\int_0^1 \frac{dx}{1+x^2}$ by using Simpson's $3/8$ th rule taking $n = 6$ and hence deduce an approximate value of π.	9	L2	CO4	PO1
	UNIT - V	**18**			
6 a.	i) Write the Regula-Falsi iteration formula. ii) Find a positive root of $x^4 - x = 10$ correct to three decimal places using Newton-Raphson method.	9	L1	CO4	PO1
b.	By using the Runge-Kutta method of order 4, find the approximate value of $y(0.5)$, given $y' = \frac{1}{x+y}$, $y(0) = 1$, taking step length $h = 0.5$.	9	L3	CO4	PO2
c.	Given $\frac{dy}{dx} - 2e^x - y$, compute y at $x = 0.4$, given $y(0) = 2, y(0.1) = 2.010, y(0.2) = 2.04, y(0.3) = 2.09$ by Milne's Predictor and Corrector method.	9	L2	CO4	PO2

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