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P.E.S. College of Engineering, Mandya - 571 401

(An Autonomous Institution affiliated to VTU, Belgaum)

Third Semester, B.E. - Computer Science and Engineering

Semester End Examination; Dec. - 2014

Discrete Mathematical Structures

Time: 3 hrs *Max. Marks: 100*

Note: *i) Answer* ***FIVE*** *full questions, selecting* ***ONE*** *full question from each Unit.*
ii) Assume suitable missing data if any.

Unit - I

1. a. In how many ways can 10 identical pencils be distributed among 5 children in the following cases;
(i) There are no restrictions — 6
(ii) Each Child gets at least one pencil
(iii) The youngest child gets at least two pencils

b. How many arrangements are there of all the letters in SOCIOLOGICAL such that
i) letter A and G are adjacent — 7
ii) are all the vowels adjacent

c. How many positive integers n can we form using the digits 3,4,4,5,5,6,7, if we want n to exceed 5,000,000? — 7

2 a. Define a set, proper subset and power set, with an example. — 6

b. The probability that a contractor will get a plumbing contract is $\frac{2}{3}$ and the probability that he will not get an electric contract is 5/9, if the probability of getting at least one contract is 4/5, what is the probability that he will get both the contracts? — 7

c. For any sets A,B,C,D, prove by using laws that $(A\cap B)\cup(A\cap B\cap\overline{C}\cap D)\cup(\overline{A}\cap B)=B$ — 7

Unit - II

3 a. Define Tautology and contradiction, prove that the proposition $[(P\to r)\wedge(q\to r)]\to[P\vee q\to r]$ is a tautology. — 7

b. Define converse, inverse and contra positive of a conditional with truth table, also state the converse, inverse and contra positive of the following statement. — 7
"If triangle is not isosceles, then it is not equilateral"

c. Prove the logical equivalence without using truth table. — 6
$(p\to q)\wedge(\neg q\wedge(r\vee\neg q))\Leftrightarrow\neg(q\vee p)$

4 a. Establish the validity of the following argument 6

$\forall x, [p(x) \vee q(x)]$

$\forall x, [\{\neg p(x) \wedge q(x)\} \rightarrow r(x)]$

$\therefore \forall x [\neg r(x) \rightarrow p(x)]$

b. if $p(x): x \geq 0$, $q(x): x^2 \geq 0$, $r(x): x^2 - 3x - 4 = 0$ 4

$S(x) = x^3 - 3 > 0$ Find truth values of

i) $\exists x [p(x) \wedge q(x)]$ ii) $\forall x [p(x) \rightarrow q(x)]$ iii) $\forall x, q(x) \rightarrow s(x)$ iv) $\forall x [r(x) \vee s(x)]$

c. Negate and simplify each of the following: i) $\forall x [p(x) \wedge \neg q(x)]$ 4

ii) $\exists x, [[p(x) \wedge q(x)] \rightarrow r(x)]$

d. Find whether the following argument is valid 6

if a triangle has two equal sides, then it is isosceles

if a triangle is isosceles, then it has two equal angles,

The triangle ABC does not have two equal angles

$\therefore$ ABC does not have two equal sides

Unit - III

5 a. Define principle of mathematical induction. Establish the following by mathematical induction $\sum_{t=1}^{n} i(2^i) = 2 + (n-1)2^{n+1}$ 7

b. Find a unique solution for the recurrence relation $4a_n - 5a_{n-1} = 0, \quad n > 1, a_0 = 1$. 6

c. Let F_n denote n^{th} Fibonacci number Prove that 7

$$\sum_{t=1}^{n} \frac{F(i-1)}{2^i} = 1 = \frac{F(n+2)}{2^n}$$

6 a. If f: A→B, g: B→C and h: C→D are three functions. Then prove that (hog)of=ho(gof) 7

b. Let f: Z→Z be defined by f(a) = a + 1 for a∈ Z show that f is a bijection. 6

c. Find the number of ways of distributing four district objects among three identical containers with some container possibly empty. 7

Unit - IV

7 a. A={1,2,3,4}, B={w, x, y, z} and C= {5,6,7} also let R_1 be a relation from A to B, defined by R_1 = {(1, x), (2, x), (3, y) (3, z)} and R_2 & R_3 be relation from B & C be defined by R_2 = {(w, 5), (x, 6)} R_3 = {(w, 5), (w, 6)} Find $R_1 0 R_2$ and $R_1 0 R_3$ 6

b. Define partially ordered set and draw and Hasse diagram of all positive divisior of 36. 7

c. A={1,2,3,4,5,6,7,8,9,10,11,12} on this set define the relation R by (x,y)∈ R if and only if (x-y) is a multiple of 5. Verify the R is an equivalence relation. 7

Contd…..3

8 a. Define least element, greatest element, minimal, maximal element of a relation R on A. 7

b. Find the number of equivalence relations that can be defined on a Finite set A with $|A| = 6$ 7

c. Let (A, R_1) and (B, R_2) be posets, on A X B, define the relation R by (a, b) R (x, y) if aR_1x and bR_2y prove that R is a partial order. 6

Unit - V

9 a. Define a group, subgroup with example. 6

b. State and prove Lagrange's theorem. 7

c. Let G be the set of all non-zero real numbers and let $a*b = \frac{1}{2}ab$, show that (G,*) is an Abelian groups. 7

10 a. Show that Z_5 is an integral domain. 6

b. Prove that the set Z with binary operations $\oplus$ and $\odot$ defined by $x \oplus y = x + y - 1$ and $x \odot y = x + y - xy$ is a commutative ring. 7

c. Write short notes on:

i) Encoding and Decoding of a message 7

ii) Hamming matrix and Generator matrix

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