

--	--	--	--	--	--	--	--	--	--



P.E.S. College of Engineering, Mandya - 571 401

(An Autonomous Institution affiliated to VTU, Belgaum)

Third Semester, B.E. - Computer Science and Engineering

Semester End Examination; Dec. - 2014

Discrete Mathematical Structures

Time: 3 hrs

Max. Marks: 100

- Note:** i) Answer **FIVE** full questions, selecting **ONE** full question from each Unit.
ii) Assume suitable missing data if any.

Unit - I

1. a. In how many ways can 10 identical pencils be distributed among 5 children in the following cases;
 - (i) There are no restrictions 6
 - (ii) Each Child gets at least one pencil
 - (iii) The youngest child gets at least two pencils
- b. How many arrangements are there of all the letters in SOCIOLOGICAL such that
 - i) letter A and G are adjacent 7
 - ii) are all the vowels adjacent
- c. How many positive integers n can we form using the digits 3,4,4,5,5,6,7, if we want n to exceed 5,000,000? 7
2. a. Define a set, proper subset and power set, with an example. 6
- b. The probability that a contractor will get a plumbing contract is $\frac{2}{3}$ and the probability that he will not get an electric contract is $\frac{5}{9}$, if the probability of getting at least one contract is $\frac{4}{5}$, what is the probability that he will get both the contracts? 7
- c. For any sets A,B,C,D, prove by using laws that $(A \cap B) \cup (A \cap B \cap \bar{C} \cap D) \cup (\bar{A} \cap B) = B$ 7

Unit - II

3. a. Define Tautology and contradiction, prove that the proposition $[(P \rightarrow r) \wedge (q \rightarrow r)] \rightarrow [P \vee q \rightarrow r]$ is a tautology. 7
- b. Define converse, inverse and contra positive of a conditional with truth table, also state the converse, inverse and contra positive of the following statement. 7
"If triangle is not isosceles, then it is not equilateral"
- c. Prove the logical equivalence without using truth table. 6
 $(p \rightarrow q) \wedge (\neg q \wedge (r \vee \neg q)) \Leftrightarrow \neg(q \vee p)$

4 a. Establish the validity of the following argument

$$\forall x, [p(x) \vee q(x)]$$

$$\forall x, [\neg p(x) \wedge q(x)] \rightarrow r(x)$$

$$\therefore \forall x [\neg r(x) \rightarrow p(x)]$$

6

b. if $p(x): x \geq 0, q(x): x^2 \geq 0, r(x): x^2 - 3x - 4 = 0$

$$S(x) = x^3 - 3 > 0$$
 Find truth values of

4

i) $\exists x [p(x) \wedge q(x)]$ ii) $\forall x [p(x) \rightarrow q(x)]$ iii) $\forall x, q(x) \rightarrow s(x)$ iv) $\forall x [r(x) \vee s(x)]$

c. Negate and simplify each of the following: i) $\forall x [p(x) \wedge \neg q(x)]$

4

ii) $\exists x, [[p(x) \wedge q(x)] \rightarrow r(x)]$

d. Find whether the following argument is valid

if a triangle has two equal sides, then it is isosceles

if a triangle is isosceles, then it has two equal angles,

6

The triangle ABC does not have two equal angles

\therefore ABC does not have two equal sides

Unit - III

5 a. Define principle of mathematical induction. Establish the following by mathematical

$$\text{induction } \sum_{i=1}^n i(2^i) = 2 + (n-1)2^{n+1}$$

7

b. Find a unique solution for the recurrence relation $4a_n - 5a_{n-1} = 0, n > 1, a_0 = 1.$

6

c. Let F_n denote n^{th} Fibonacci number Prove that

$$\sum_{i=1}^n \frac{F(i-1)}{2^i} = 1 = \frac{F(n+2)}{2^n}$$

7

6 a. If $f: A \rightarrow B, g: B \rightarrow C$ and $h: C \rightarrow D$ are three functions. Then prove that $(hog)of = ho(gof)$

7

b. Let $f: Z \rightarrow Z$ be defined by $f(a) = a + 1$ for $a \in Z$ show that f is a bijection.

6

c. Find the number of ways of distributing four distinct objects among three identical containers with some container possibly empty.

7

Unit - IV

7 a. $A = \{1,2,3,4\}, B = \{w, x, y, z\}$ and $C = \{5,6,7\}$ also let R_1 be a relation from A to B , defined by

$$R_1 = \{(1, x), (2, x), (3, y), (3, z)\}$$
 and R_2 & R_3 be relation from B & C be defined by

6

$$R_2 = \{(w, 5), (x, 6)\} \quad R_3 = \{(w, 5), (w, 6)\}$$
 Find $R_1 \circ R_2$ and $R_1 \circ R_3$

b. Define partially ordered set and draw and Hasse diagram of all positive divisor of 36.

7

c. $A = \{1,2,3,4,5,6,7,8,9,10,11,12\}$ on this set define the relation R by $(x,y) \in R$ if and only if $(x-y)$ is a multiple of 5. Verify the R is an equivalence relation.

7

- 8 a. Define least element, greatest element, minimal, maximal element of a relation R on A. 7
- b. Find the number of equivalence relations that can be defined on a Finite set A with $|A| = 6$ 7
- c. Let (A, R_1) and (B, R_2) be posets, on $A \times B$, define the relation R by $(a, b) R (x, y)$ if aR_1x and bR_2y prove that R is a partial order. 6

Unit - V

- 9 a. Define a group, subgroup with example. 6
- b. State and prove Lagrange's theorem. 7
- c. Let G be the set of all non-zero real numbers and let $a * b = \frac{1}{2} ab$, show that $(G, *)$ is an Abelian groups. 7
- 10 a. Show that Z_5 is an integral domain. 6
- b. Prove that the set Z with binary operations \oplus and \square defined by $x \oplus y = x + y - 1$ and $x \square y = x + y - xy$ is a commutative ring. 7
- c. Write short notes on: 7
- i) Encoding and Decoding of a message
 - ii) Hamming matrix and Generator matrix

* * * * *