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P.E.S. College of Engineering, Mandya - 571 401

(An Autonomous Institution affiliated to VTU, Belgaum)

Third Semester, B.E. - Computer Science and Engineering

Semester End Examination; Dec. - 2015

Discrete Mathematical structures

Time: 3 hrs *Max. Marks: 100*

Note: *Answer any **FIVE** full questions, selecting **ONE** full question from each unit.*

UNIT - I

1 a. For $u=\{1,2,3,..........9,10\}$ let $A=\{1,2,3,4,5\}$, $B=\{1,2,4,8\}$, $C=\{1,2,3,5,7\}$ and $D=\{2,4,6,8\}$ Determine the following (i) $(A\cup B)\cap C$ (ii) $\overline{C}\cup\overline{D}$ (iii) $(A\cup B)-C$ (iv) $(B-C)-D$ (v) $(A\cup B)-(C\cap D)$. 5

b. Juan tosses a fair coin five times. What is the probability the number of heads always exceeds the number of tails as each outcome is observed? 5

c. Determine the coefficient of $x^2y^2z^3$ in the expansion of $(3x-2y-4z)^7$ 4

d. In how many ways can 10 identical pencils be distributed among 5 children in the following cases:

(i) There are no restrictions (ii) Each child gets atleast one pencil (iii) The youngest child gets atleast two pencils. 6

2 a. The board of directors of a pharmaceutical corporation has 10 members. An upcoming stock holders' meeting is scheduled to approve a new slate of company officers (chosen from the 10 board members).

(i) How many different slates consisting of a president, vice president, secretary and treasurer can be the board present to the stock holders for their approval? 6

(ii) Three members of the board of directors are physicians. How many slates from part (i) have (A) a physician nominated for the presidency?

(B) Exactly one physician appearing on the slate?

b. Over the internet data are transmitted in structured blocks of bits called datagrams.

(i) In how many ways can the letters in DATAGRAM be arranged? 4

(ii) For the arrangements of part (i) how many have all three 'A' s together?

c. Using the laws of set theory, simplify the expression $\overline{\overline{(A\cup B)\cap C}\cup\overline{B}}$ 3

d. A manufacturer of 2000 automobile batteries is concerned about defective terminals and defective plates. If 1920 of batteries have neither defect, 60 have defective plates, and 20 have both defects, how many batteries have defective terminals? 7

Contd….2

UNIT - II

3 a. Negate the following and simplify the resulting statement 6

$P \wedge (Q \vee r) \wedge (\neg P \vee \neg Q \vee r)$

b. Using the laws of logic simplify the given statement 6

$[(P \to Q)] \wedge [\neg Q \wedge (r \vee \neg Q)]$

c. Establish the validity of the following argument 8

$p \wedge q$

$P \to (r \wedge Q)$

$r \to (s \vee t)$

$\underline{\neg s}$

$\therefore t$

4 a. Prove that the following argument is valid wherein C is specified element of the universe. 6

$\forall x, [P(x) \to Q(x)]$

$\forall x, [Q(x) \to r(x)]$

$\underline{\neg r(c)}$

$\therefore \neg P(c)$

b. Convert the following statement to symbolic form and also write its negation. 6

"For all *x*, if x is odd then x^2-*1* is even"

c. Give (i) a direct proof (ii) an indirect proof and (iii) proof by contradiction, for the following statement: 8

" If n is an odd integer, then n+11 is an even integer"

UNIT - III

5 a. If n is a positive integer, prove that $1-2+2-3+3-4+........+n(n+1) = \frac{n(n+1)(n+2)}{3}$ using mathematical induction. 7

b. i) For $n \geq 2$ any sets $A_1, A_2, ..., A_u \subseteq_u$ Prove that $\overline{A_1 \cup A_2 \cup .... \cup A_n} = \overline{A}_1 \cap \overline{A_2} \cap ..... \cap \overline{A}_n$

ii) The Fibonacci numbers are defined recursively by 7

$F_0 = 0, F_1 = 1$ *and* $F_n = F_{n-1} + F_{n-2}$ *for* $n \geq 2$. Evaluate F_2, F_5, F_7

c. If $A = \{1,2,3,4\}, B = \{2,5\}$ *and* $C = \{3,4,7\}$ determine 6

$A \times B;\ B \times A;\ A \cup (B \times C);\ \ (A \cup B) \times C;\ \ (A \times C) \cup (B \times C)$

6 a. For each of the following functions $g : R \to R$ determine whether the function is one-to-one and whether it is onto. If the function is not onto, determine the range $g(R)$. 4

a) $g(x) = x + 7$ b) $g(x) = 2x - 3$ c) $g(x) = x^2$ d) $g(x) = x^2 + x$

Contd...3

b. Let $S=\{3,7,11,15,19,..........95,99,103\}$, How many elements must we select from S to insure that there will be atleast two whose sum is 110? 4

c. i) Let $f,g:R\to R$ where $g(x)=1-x+x^2$ and $f(x)=ax+b$. If $(gof)(x)=9x^2-9x+3$ determine a, b.

ii) Given $P=\begin{bmatrix}1 & 2 & 3 & 4 & 5 & 6\\ 2 & 4 & 3 & 1 & 5 & 6\end{bmatrix}$ Compute p^{-1} *and* p^2. 8

d. Let $f:A\to B$ *and* $g:B\to C$ Prove that if f and g are one-to-one then gof is one-to-one. 4

UNIT - IV

7 a. Let $A=\{1,2,3,4\}$ *and* $R=\{(1,1),(1,2),(2,1),(2,2),(3,1)(3,3),(1,3)(4,1),(4,4)\}$ be a relation on A. Is R an equivalence relation? 6

b. Draw the Hasse diagram for factors of 36. 8

c. Let $A=\{1,2,3,4,6\}$ and R be a relation on A defined by aRb if and only if a is a multiple of b. Represent the relation R as a matrix and draw its digraph. 6

8 a. Consider the following relation on the set $A=\{1,2,3\}:R_1=\{(1,1)(1,2),(1,3),(3,3)\}$ and $R_2=\{(1,1),(1,2),(2,1),(2,2),(3,3)\}$ which of these are 6

(i) reflexive (ii) symmetric (iii) transitive (iv) antisymmetric

b. The digraph for a relation on set $A=\{1,2,3,4\}$ is as shown in fig 8(b)

(i) Verify that (A, R) is a poset and find its Hasse diagram.

(ii) Topological sort (A, R) 8

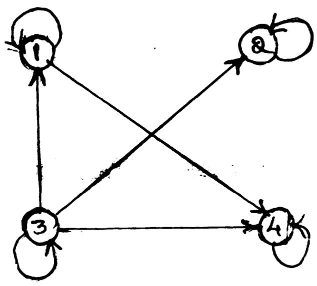

Fig Q8(b)

c. For the posets shown in the following Fig Q8(c), find (i) all upper bounds (ii) all lower bounds and (iii) LUB and GLB of the set B, where $B=\{3,4,5\}$ 6

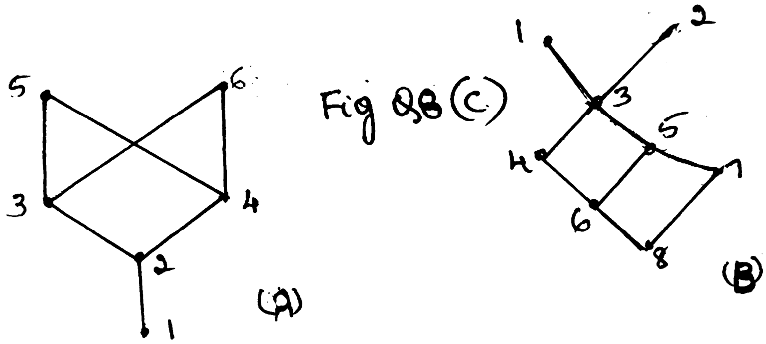

Fig Q8(c)

UNIT - V

9 a. What is cyclic group? Explain and hence, show that the group (G,*) whose multiplication table is as given below is cyclic 6

*	a	b	c	d	e	f
a	a	b	c	d	e	f
b	b	c	d	e	f	a
c	c	d	e	f	a	b
d	d	e	f	a	b	c
e	e	f	a	b	c	d
f	f	a	b	c	d	e

b. Explain Lagrange's theorem. If G is a group of order n, and $a \in G$, Prove that $a^n = e$ 6

c. Define the following terms with respect to coding theory
(i) Parity check code (ii) Hamming distance (iii) Group code (iv) Generator matrix. 8

10a. The parity –check matrix for an encoding function $Z_2^3 - 7Z_2^6$ is given by

$$H = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$ 6

(i) Determine the associated generator matrix (ii) Does this code correct all single errors in transmission?

b. a is a group and $a, b \in G$. Prove that (i) $\left(a^{-1}\right) - 1 = a$ and (ii) $\left(ab\right)^{-1} = b^{-1}a^{-1}$ 6

c. Define sub group. If H, K are subgroup or G. Grove that H∩K is also subgroup. 8

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