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P.E.S. College of Engineering, Mandya - 571 401

(An Autonomous Institution affiliated to VTU, Belgaum)

Third Semester, B.E. - Computer Science and Engineering

Semester End Examination; Dec - 2016/Jan - 2017

Discrete Mathematical Structures

Time: 3 hrs *Max. Marks: 100*

***Note**: Answer **FIVE** full questions, selecting **ONE** full question from each unit.*

UNIT - I

1 a. How many positive integers n can we form using the digits 3, 4, 4, 5, 5, 6, 7, if we want n to exceed 4000000? 6

b. How many ways can a gambler draw five cards from a standard deck and get, 8

i) five cards of the same suit? ii) four aces

iii) four of a kind? iv) three aces and two jacks?

c. Determine the number of integer solutions of, 6

$x_1 + x_2 + x_3 + x_4 = 32$

where, i) $x_i \geq 0, \ 1 \leq i \leq 4$

ii) $x_i > 0, \ 1 \leq i \leq 4$

iii) $x_{1,} x_2 \geq 5, x_3, x_4 \geq 7$.

2 a. Let $U = \{1,2,3,9,10\}$, $A = \{1,2,3,4,5\}$, $B = \{3,4,5,6,7\}$ and $C = \{7,8,9\}$. Find 5

i) $A \cap B$ ii) $A \cup B$ iii) $B \cap C$ iv) $A \cap C$ v) $A \Delta B$.

b. Prove that $(A \cap B) \cup \left[B \cap \left((C \cap D) \cup \left(C \cap \overline{D}\right)\right)\right] = B \cap (A \cup C)$. 7

c. In a survey of 120 passengers, an airline found that 48 enjoyed wine with their meals, 78 enjoyed mixed drinks and 66 enjoyed iced tea. In addition 36 enjoyed any and given pair of these beverages and 24 passengers enjoyed them all. If two passengers are selected at random from the survey sample of 120 what is the probability that, 8

i) They both want only iced tea with their meals?

ii) They both enjoy exactly two of the three beverage offerings?

UNIT - II

3 a. Verify that $\left[P \rightarrow (q \rightarrow r)\right] \rightarrow \left[(p \rightarrow q) \rightarrow (p \rightarrow r)\right]$ is a tautology. 6

b. Provide the steps and reasons to establish the following logical equivalences, 6

$(P \rightarrow q) \wedge \left[\neg q \wedge (r \vee \neg q)\right] \Leftrightarrow \neg(q \vee p)$

Contd......2

c. Establish the validity of the following argument by expression in symbolic form "If the band could not pay rock music or the refreshments were not delivered on time then the New year's party would have been canceled and Alicia would have been angry. If the party were canceled, then refunds would have had to be made. No refunds were made. Therefore the band could play rock music". 8

4 a. For the universe of all integers let $p(x), q(x), r(x), s(x)$ *and* $t(x)$ is defined as,

$P(x): x > 0$
$q(x):$ is even
$r(x): x$ is a perfect square
$s(x): x$is divisible by 4
$t(x): x$ is divisible by 5

Write the following statements in symbolic form and also find whether each of the statements is true or false. 8

i) At least one integer is even
ii) There exist a positive integer that is even
iii) If x is even, then x is not divisible by 5
iv) No even integer is divisible by 5.

b. Provide the steps and reasons to establish the validity of the argument, 6

$\forall x\left[p(x) \vee q(x)\right]$
$\forall x\left[\left(\neg p(x) \wedge q(x)\right) \rightarrow r(x)\right]$
$\therefore \forall x\left[\neg r(x) \rightarrow p(x)\right].$

c. For all integers k and l, if k, l are both odd, then $k+l$, is even. 6

UNIT - III

5 a. If n is a positive integer then prove that, 6

$$1.2 + 2.3 + 3.4........ + n(n+1) = \frac{n(n+1)(n+2)}{3}.$$

b. Define Binomial coefficient recursively and hence define Eulerian numbers $a_m, k, m, k \in z$ recursively and also fine the value of $a_m, k,$ where$1 \le m \le 5$ *and* $0 \le k \le m-1$. 8

c. For any $A = \{1,2,3\}, B = \{2,4,5\}$,determine the following; 6

i) $|A \times B|$ ii) the number of relations from A to B iii) the number of relations on A

iv) the number of relations from A to B that contain $(1,2)$ and $(1,5)$

v) the number of relations from A to B that contain exactly five ordered pairs

vi) the number of relations on A that contain at least seven elements

Contd......3

6 a. Prove that, $i)\ \lfloor 2x \rfloor = \lfloor x \rfloor + \lfloor x + \frac{1}{2} \rfloor$ for any real x 6

$ii)\ \lfloor x \rfloor = \lceil x \rceil - 1$ for any $x \in R - Z$.

b. Let $S \subset Z^{+}$ where $|S| = 25$, then prove that S contains two elements that have the same remainder upon division by 24. 6

c. Let $f : R \to R$ be defined by,

$$f(x) = \begin{cases} 3x-5 & x > 0 \\ -3x+1 & x \le 0 \end{cases}$$

i) Determine : $f(1),\ f(-1),\ f\left(5/3\right)$ 8

ii) Find $f^{-1}(0),\ f^{-1}(1),\ f^{-1}(-3)$

iii) What are $f^{-1}([-5,5])$, and $f^{-1}([-6,5])$.

UNIT - IV

7 a. Prove that the number of Symmetric relation on set A is $2^{\frac{1}{2}}(n^2+n)$. 5

b. If $A = \{1,2,3,4\}$ give an example of relation R on A that is,

i) Reflexive and Symmetric but not transitive 6

ii) Reflexive and transitive but not symmetric

iii) Symmetric and transitive but not reflexive.

c. Let $A = \{1,2,3,4,5,6,7\}$ How many symmetric relations on A contain exactly, 4

i) four ordered pairs ii) five ordered pairs.

d. Let $A = \{1,2,3,6,9,18\}$ and define R on A by xRy if x/y Verify it is Poset? Draw Hasse diagram. 5

8 a. Define R on $A = \{1,2,3,4,5,6,7,8,9,10,11,12\}$ by $(x,y) \in R$ if $x - y$ is a multiple of 5,

i) Prove that R is an equivalence relation on A 6

ii) Determine the equivalence classes and portion of A induced by R.

b. State Topological sorting algorithm. 6

c. Let $A = \{1,2,3,,5,30\}$,

i) Prove that $(A,1)$ is a lattice by constructing a meet joint table.

ii) Prove that **.** is not distributive over + in this lattice by identifying elements a, b, c in A for which $a.(b+c) \neq a.b + a.c$ 8

iii) Prove that + is not a distributive over **.** by showing that $a + (b.c) \neq (a+b).(a+c)$ for some elements a, b, c in A.

Contd......4

UNIT - V

9 a. Show that $(A, .)$ is an abelian group where $A = \{a \in Q | a \neq -1\}$ and or any $a, b \in A$, $a.b = a + b + ab$ 8

b. Let G be the group of complex numbers $\{1, -1, i, -i\}$ under multiplication. Construct multiplication table for this group with $H = (Z_4, +)$ consider $f : G \to H$ defined by $f(1) = [0]$, $f(-1) = [2]$, $f(i) = [1]$ $f(-i) = [3]$. 7

c. For $B = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 1 & 2 & 3 \end{pmatrix}$ Find the subgroup $K = \langle B \rangle$. 5

10 a. Let C be a set of code words, where $C \subseteq Z_2^7$. In each of the following, two of e(error pattern), r(received word) and C (code word) are given, with $r = C + e$. Determine the third term, 6

i) $C = 1010110$ $r = 1011111$

ii) $C = 1010110$ $e = 0101101$

iii) $e = 0101111$ $r = 0000111$.

b. Let $H = \begin{bmatrix} 1 & 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$ be the parity check for a Hamming $(7, 4)$ code, 8

i) Encode the following messages,

1000 1100 1011 1110

ii) Decode the following received words,

1100001 1110111 0010001 0011100

Using the following table;

Syndrome	000	001	010	100
Coset Leader	0000000	0000001	0000010	0000100
Syndrome	011	101	110	111
Coset Leader	0010000	0100000	1000000	0001000

c. I) For $a(2, 6)$ triple repetition code, Find: 6

i) $E(10)$ ii) $E(001010)$ iii) $D(101010)$.

II) For a $(3, 9)$ triple repetition code, Find:

i) $E(100)$ ii) $E(011)$ iii) $D(100100110)$.

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