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P.E.S. College of Engineering, Mandya - 571 401

(An Autonomous Institution affiliated to VTU, Belgaum)

Fourth Semester, B.E. - Computer Science and Engineering

Semester End Examination; June/July - 2015

Graph Theory and Combinatorics

Time: 3 hrs *Max. Marks: 100*

***Note**: i) Answer **FIVE** full questions, selecting **ONE** full question from each **Unit**.*
ii) Assume suitable missing data if any.

UNIT - I

1. a. Define graph isomorphism. Give an example of isomorphic graphs. 5

b. Let G be an undirected graph with no isolated vertices. Then prove that G has an Euler circuit iff G is connected and every vertex in G has even degree. 10

c. Write a short notes on Konigsberg bridge problem. 5

2 a. Let D be a digraph with an odd number of vertices. Prove that if each vertex of D has an odd – degree then D has an odd number of vertices with odd in – degrees. 5

b. If G(V, E) is a simple graph, prove that
$2\,|E| \leq |V|^2 - |V|$ 5

c. How many vertices and how many edges are there in the complete bipartite graph $K_{4,7}$ and $K_{7,11}$? 5

d. Show that if a bipartite graph $G(V_1\, V_2, E)$ is regular, then $|V_1| = |V_2|$. 5

UNIT – II

3 a. Show that the complete graph K_5 is a non planar graph. 5

b. Prove that a connected planar graph G with n vertices and m edges has exactly m – n + 2 regions in all of its diagrams. 10

c. Write the five steps of elementary reduction method for detection of planarity. 5

4. a. Prove that the sum of the degrees of the regions of a planar graph G o is equal to twice the number of edges in G. 5

b. Prove that a graph G is 2 - chromatic if an only if it is a non – null bipartite graph. 5

c. Prove the following :
i) for any graph G, the constant term in $P(G, \lambda)$ is zero 5
ii) for any graph G(V, E) with $|E| \geq 1$, the sum of the coefficients in $P(G, \lambda)$is zero

d. Find the chromatic number and the chromatic polynomial for the graph $K_{1,n.}$ 5

Contd…2

UNIT - III

5. a. If a tree T has four vertices of degree 2, one vertex of degree 3, two vertices of degree 4 and one vertex of degree 5 find the number of leaves in T. 5

b. Write the preorder and post order traversal of the tree shown below. 5

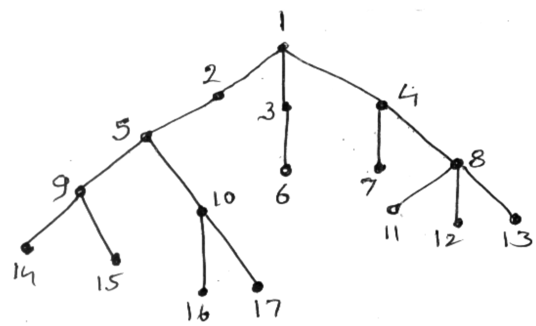

Fig 5(b)

c. Apply merge sort to the list 5
-1, 7, 4, 11, 5, -8, 15, -3, -2, 6, 10, 3

d. Prove that a graph is connected if and only if it has a spanning tree. 5

6. a. Write the steps of DFS algorithm. 5

b. Construct an optimal prefix code for the symbols a, o, q, u, y, z that occur with frequencies 20, 28, 4, 17, 12, 7 respectively. 5

c. Write Kruskal's algorithm. 5

d. For the network shown below, determine the maximum flow between A & Z by identifying a cut – set of minimum capacity. 5

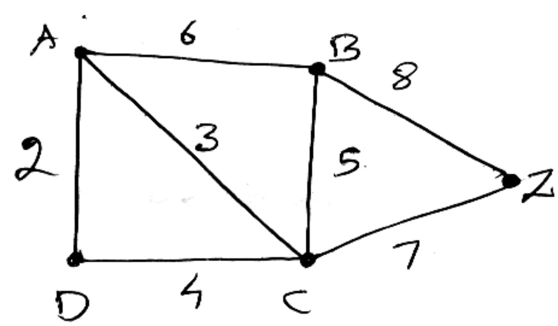

Fig 6(d)

UNIT - IV

7. a. i) Find the number of binary sequences of length n. 5
ii) Find the number of binary sequences of length n that contain an even number of 1's.

b. How many positive integers n can we form using the digits 3, 4, 4, 5, 5, 6, 7 if we want n to exceed 5,000,000. 5

Contd…3

c. Certain question paper contains three parts A, B, C with four questions in Part-A, five questions in Part-B and six questions in part-C It is required to answer seven questions selecting at least two questions from each part. In how many different ways can a student select his seven questions? 5

d. Find the term which contains x^{11} and y^4 in the expansion of $(2x^3-3xy^2+z^2)^6$. 5

8\. a. Determine the number of positive integers n such that $1 \leq n \leq 100$ and n is not divisible by 2, 3 or 5. 5

b. In how many ways can the integers 1, 2, 3, ---- 10 be arranged in a line so that no even integer is in its natural place. 5

c. Find the rook polynomial for the 3 X 3 board by using the expansion formula. 5

d. Using generating function, find the number of partitions of n = 6. 5

UNIT - V

9 a. Suppose there are $n \geq 2$ persons at a party and that each of these persons shakes hands with all of the other persons present. Using a recurrence relation find the number of hand shakes. 5

b. Solve the recurrence,

$F_{n+2} = F_{n+1} + F_n$, $n \geq 0$, $F_0 = 0$, $F_1 = 1$. 5

c. Solve the recurrence

$a_n + a_{n-1} - 8a_{n-2} - 12a_{n-3} = 0, n \geq 3$ with $a_0 = 1, a_1 = 5, a_2 = 1$ 5

d. Solve the recurrence

$a_n + 4a_{n-1} + 4a_{n-2} = 8$ for $n \geq 2$ and $a_0 = 1, a_1 = 2$ 5

10a. Solve the recurrence relation

$a_n + 2a_{n-1} - 3a_{n-2} = 4n^2 - 5$ for $n \geq 2$. 10

b. Using the generating function method, solve the recurrence

$a_n = 3a_{n-1} = n, n \geq 1$. 10

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