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P.E.S. College of Engineering, Mandya - 571 401

(An Autonomous Institution affiliated to VTU, Belgaum)

Fourth Semester, B.E. – Computer Science and Engineering
Semester End Examination; June - 2016
Graph Theory and Combinatorics

Time: 3 hrs *Max. Marks: 100*

***Note**: Answer **FIVE** full questions, selecting **ONE** full question from each unit.*

UNIT - I

1 a. Define graph isomorphism. Give an example of isomorphic graphs. 5

b. Write a short note on Konigsberg bridge problem. 5

c. In the complete graph with *n* vertices, where n is an odd number ≥3. There are $\frac{(n-1)}{2}$ edge – disjoint Hamiltonian cycles. 10

2 a. If G = G(V, E) is a simple graph, prove that $2|E| \leq |V|^2 - |V|$ 5

b. In every graph, the number of vertices of odd degrees is even. 5

c. For a graph with *n* vertices and *m* edges, if δ is the minimum and Δ is the maximum of the degrees of vertices, show that $\delta \leq \frac{2m}{n} \leq \Delta$ 5

d. Prove that two simple graph G_1 and G_2 are isomorphic if and only if their complements $\overline{G_1}$ *and* $\overline{G_2}$ are isomorphic. 5

UNIT - II

3 a. Show that the complement graph K_5 is a non planar graph. 5

b. Prove that a connected planar graph G with *n* vertices and *m* edges has exactly *m* – *n* + 2 regions in all of its diagrams. 10

c. Prove that a tree with *n* vertices has *n -1* edges 5

4 a. Show that the complete graph $K_{3,3}$ namely, the Kuratowski's second graph is a non planar graph. 5

b. Let G be a connected simple planar graph with fewer than 12 regions in which each vertex has degree atleast 3. Prove that G has a region bounded by at most four edges. 10

c. Prove that every connected simple planar graph G is 6-colorable. 5

UNIT - III

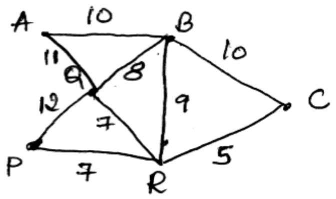
5 a. Apply merge sort to the lift -1, 7, 4, 11, 5, -8, 15, -3, -2, 6, 10, 3. 5

b. Explain DFS algorithm with each step involved in it. 5

Contd….2

c. Construct an optimal prefix code for the symbols a, o, q, u, y, z that occur that frequencies 20, 28, 4, 17, 12, 7 respectively. 5

d. Using Kruskal's algorithm, find a minimal spanning tree for the weighted graph shown below, 5



6 a. Write Kruskal's algorithm. 5

b. For the network shown below, determine the maximum flow between the vertices A and D by identifying the cut-set of minimum capacity. 5

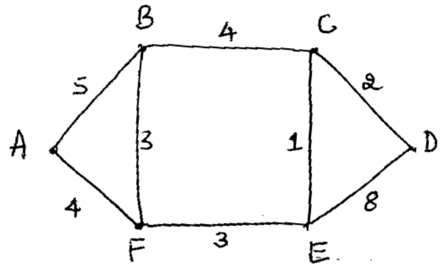
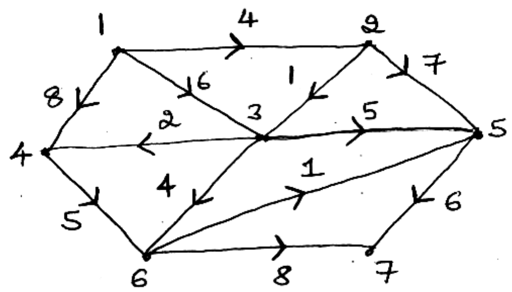


Fig. 6b

c. Using the Dijkstra's algorithm, obtain the shortest path from vertex 1 to each of the other vertices in the weighted, directed network shown in the figure. Indicate the weights of their shortest paths. 10



UNIT - IV

7 a. (i) Find the number of binary sequences of length *n*.

(ii) Find the member of binary sequences of length '*n*' that contain an even number of 1′s. 5

b. Prove the identity: $C(n,r).C(r,k) = C(n,k).C(n-k,r-k),\ for\ n \geq r \geq k$ 5

c. A total amount of Rs.1500 is to be distributed to 3 poor students P, Q, R of a class. In how many ways the distribution can be made in multiples of Rs.100,

(i) If everyone of there must get atleast Rs.300?

(ii) If P must get atleast Rs.500, Q and R must get atleast Rs.400 each. 5

d. Determine the number of positive integers *n* such that $1 \leq n \leq 100$ and n is not divisible by 2, 3, or 5. 5

8 a. There are eight letters to eight different people to be placed in eight different addressed envelopes. Find the number of ways of doing this so that atleast one letter gets to right person. 5

b. Find the rook polynomial for the 2x2 board by using the expansion formula. 5

c. Obtain a generating function for the sequence $\langle ar \rangle$ where $a_r = 0$ for $0 \le r \le n-1$, and $a_r = \binom{r-1}{n-1}$ for $r \ge n$ 5

d. Using generating function, find the number of partitions of $n = 6$. 5

UNIT - V

9 a. The number of virus affected files in a system is 1000 (to start with) and this increases 250% every two hours. Use the recurrence relation to determine two hours. Use the recurrence relation to determine the number of virus affected files in the system after one day. 5

b. Solve the recurrence relation $a_n + a_{n-1} - 6a_{n-2} = 0$ for $n \ge 2$ that $a_0 = -1$ *and* $a_1 = 8$ 5

c. Find and solve a recurrence relation for the number of binary sequences of length $n \ge 1$ that has no consecutive 0's. 10

10 a. Solve the recurrence relation,

$a_n + a_{n-1} - 8a_{n-2} - 12a_{n-3} = 0, \quad n \ge 3 \text{ with } a_0 = 1, \quad a_1 = 5, \quad a_2 = 1$ 5

b. Solve the recurrence relation $a_{n+2} - 10a_{n+1} + 21a_n = 3n^2 - 2, n \ge 0$ 5

c. Find a generating function for recurrence relation,

$a_{n+1} - a_n = n^2, n \ge 0 \text{ and } a_0 = 1$ Hence solve it. 10

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