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P.E.S. College of Engineering, Mandya - 571 401

(An Autonomous Institution affiliated to VTU, Belgaum)

Fourth Semester, B.E. - Computer Science and Engineering

Make - up Examination; July - 2016

Graph Theory and Combinatorics

Time: 3 hrs *Max. Marks: 100*

***Note**: i) Answer **FIVE** full questions, selecting **ONE** full question from each unit.*
ii) Assume suitable missing data if any.

UNIT - I

1	a.	Define the following : i) Finite Graph ii) Infinite Graph iii) Subgraph iv) Complement of a graph v) Graph Isomorphism.	7
	b.	Explain Konigsberg bridge problem. Also explain why it has no solution?	8
	c.	Describe the TSP problem. How it is connected with the Hamiltonian circuits?	5
2	a.	Define Euler and Hamiltonian graph with an example for each.	6
	b.	Prove that in a complete graph with n vertices there are $(n-1)/2$ edge disjoint Hamiltonian Circuits, if n is an odd number ≥ 3.	10
	c.	If G = G (V, E) is a simple graph, prove that $2\lvert E\rvert \leq \lvert V\rvert^2 - \lvert V\rvert$.	4

UNIT - II

3	a.	Show that the complete graph K_5 is a non-planar graph.	5
	b.	Show that a connected planar graph G with n vertices and m edges has exactly $m-n+2$ regions in all of its diagrams.	10
	c.	Let G be a 4- regular connected planar graph having 16 edges. Find the number of regions of G.	5
4	a.	Let G be a connected planar graph, with n vertices, m edges and r regions and let its dual G* have n^* vertices, m^*edges and r^* regions. Then show that $n^* = r$, $m^* = m$, $r^* = n$.	5
	b.	Find the chromatic number and the chromatic polynomial for the graph $K_{1,n}$.	5
	c.	Let G = G(V, E) and G′ = G′ (V′, E′) be two graphs and $f:G \to G'$ be an isomorphism, prove the following : i) $f^{-1}:G' \to G$ is also an isomorphism ii) For any vertex v in G, deg(v) in G = deg (f(v)) in G′.	10

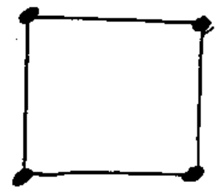
UNIT - III

5	a.	Prove that a tree with n vertices has $n-1$ edges.	8
	b.	Prove that a tree with two or more vertices contains at least two leaves (Pendant vertices).	6
	c.	Prove that a graph with n vertices, $n-1$ edges and no cycles is connected.	6

Contd……2

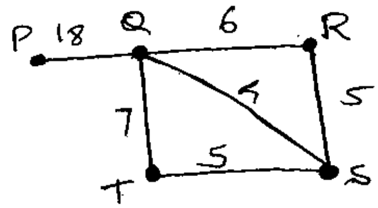
6 a. Using the merge-sort method sort the list 7, 3, 8, 4, 5, 10, 6, 2, 9. 5

b. What is a spanning tree? Find all the spanning trees of the graph shown below, 5



c. Write the steps of Kruskal's algorithm. 5

d. Find the maximum flow possible between the vertices P and S in the network given below : 5



UNIT - IV

7 a. Find the number of 3-digit even numbers with no repeated digits. 5

b. How many positive integers *n* can we form using the digits 3, 4, 4, 5, 5, 6, 7, if we want *n* to exceed 5,000,000? 5

c. A certain question paper contains three parts A, B, C with four questions in Part A, Five questions in Part B and six questions in Part C. It is required to answer seven questions selecting atleast two questions from each part. 5

In how many different ways can a student select his seven questions for answering?

d. Among the students in a hostel, 12 students study mathematics (A), 20 study physics (B), 20 study chemistry (C), and 8 study biology (D). There are 5 students for A and B, 7 students for A and C, 4 students for A and D, 16 students for B and C, 4 students for B and D, and 3 students for C and D. There are 3 students for A, B, and C, 2 for A, B and D, 2 for B, C and D, 3 for A, C and D. Finally there are 2 who study all of these subjects. Furthermore there are 71 students who do not study any of these subjects. Find the total number of students in the hostel. 5

8 a. Find the number of dearrangements of 1, 2, 3, 4. Also write all the dearrangements. 5

b. Find the rook polynomial for the 2 x 2 board for using the expansion formula. 5

c. Find a generating function for each of the following sequences : 5

i) $1^2, 2^2, 3^2$…..

ii) $0^3, 1^3, 2^3, 3^3$……

d. Using generating function, find the number of partitions of *n* = 6. 5

UNIT - V

9 a. The number of virus affected files in a system is 1000 (to start with) and this increases 250% every two hours. Use a recurrence relation to determine the number of virus affected files in the system after one day. 5

b. Solve the recurrence relation,

$a_n + a_{n-1} - 6a_{n-2} = 0 \quad for\ n \geq 2$ 5

Given that $a_0 = -1\ and\ a_1 = 8$

c. Solve the recurrence relation $2a_{n+3} = a_{n+2} + 2a_{n+1} - a_n,\ for\ n \geq 0\ with\ a_0 = 0,\ a_1 = 1\ and\ a_2 = 2.$ 5

d. Solve the recurrence relation $a_n + 4a_{n-1} + 4a_{n-2} = 8\ for\ n \geq 2\ \&\ a_0 = 1,\ a_1 = 2.$ 5

10 a. Solve the recurrence relation $a_{n+2} - 2a_{n+1} + a_n = 2^n,\ n \geq 0\ \ \&\ \ a_0 = 1,\ a_1 = 2$, by the method of generating function. 10

b. Suppose there are $n \geq 2$ persons at a party and that each of these persons shake hands with all of the other persons present. Using a recurrence relation, find the number of hand shakes. 5

c. Solve the recurrence relation, 5

$a_n = 3a_{n-1} - 2a_{n-2}\ \ for\ n \geq 2\ \ given\ that\ a_1 = 5\ and\ a_2 = 3.$

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