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P.E.S. College of Engineering, Mandya - 571 401

(An Autonomous Institution affiliated to VTU, Belgaum)

Fifth Semester, B.E. - Computer Science and Engineering

Semester End Examination; Dec. - 2014

Graph theory and Combinatorics

Time: 3 hrs *Max. Marks: 100*

***Note**: Answer any **FIVE** full questions, selecting at least **TWO** full questions from each part.*

PART - A

1. a. Define the following with an example for each: 8

   i) Graph ii) Walk iii) Circuit iv) Path

   b. In an undirected graph G = (V, E) with $|V| = v$ and $|E| = e$ and no loops, show that $\sum_{v=0}^{n} \deg(v) = 2|E|$ 4

   c. Define graph isomorphism with an example. 4

   d. Define Euler's circuit, with an example. 4

2 a. Let G = (V, E) be a loop-free connected planar graph with $|V| = v$, $|E| = e > 2$ and r regions. Then prove $3r \leq 2e$ and $e \leq 3v - 6$ 4

   b. Show that $K_{3,3}$ is non planar. 4

   c. Give an example of a connected graph that has

   i) neither an Euler circuit nor a Hamilton cycle

   ii) An Euler circuit but no Hamilton cycle

   iii) An Euler path but no Euler circuit 6

   iv) A Hamilton cycles but no Euler circuit.

   v) Both a Hamilton cycle and an Euler circuit.

   vi) Has a Hamilton path but not Hamilton circuit.

   d. Construct a coloring of the graph shown below. What is the chromatic number of the graph? 6

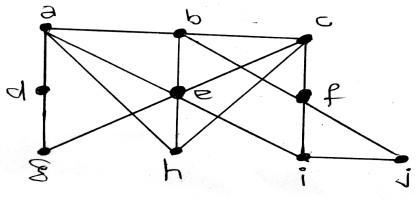


fig 2(d)

3 a. Define a tree. Prove that for every tree T = (V, E), if $|v| \geq 2$ then T has at least 2 pendent vertices. 8

   b. Write the preorder and post-order listing of the following tree. 4

Contd.....2

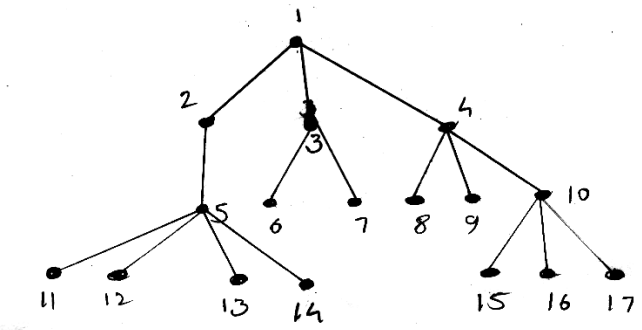
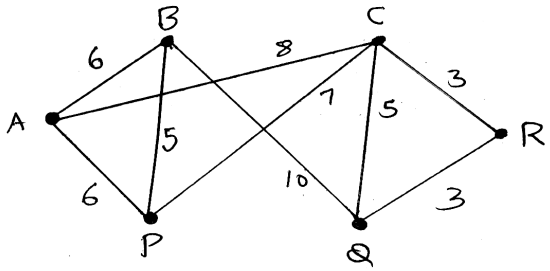


Fig 3(b)

c. Write the Depth first search algorithm. 4

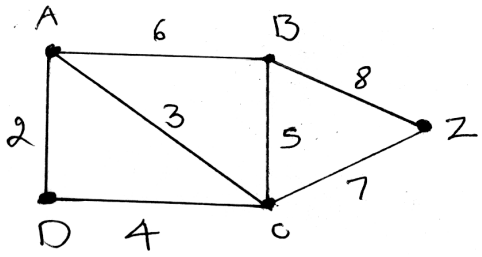
d. Construct an optimal prefix code for symbols a, o, q, u, y, z that occur with frequencies 20, 28, 4, 17, 12, 7 respectively. 4

4 a. Using the Kurskal's algorithm, find a minimal spanning tree of the weighted graph shown below.



6

b. For the network shown below, determine the maximum flow between A and Z identifying a cut-set of minimum capacity.



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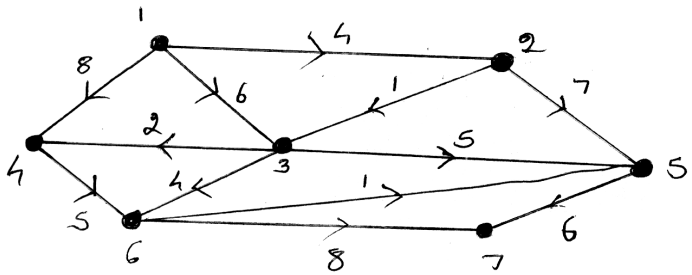
c. Using the Dijkstra's algorithm, obtain the shortest path from vertex 1 to each of the other vertices in the weighted, directed network shown below; indicate the weights of these shortest paths.



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Contd.....3

PART - B

5 a. A committee of eight is to be formed from16 men and 10 women. In how many ways can the committee be formed if:
i) there are no restrictions ii) there must be 4 men and 4 women
iii) there should be an even number of women iv) more women than men
v) at least 6 men — 10

b. How many different arrangements of the letters in TALLAHASSEE is possible? How many of these arrangements have no adjacent A's? — 4

c. Find the coefficient of $a^2b^3c^2d^5$ in the expansion of $(a + 2b - 3c + 2d + 5)^{16}$. — 6

6 a. Determine the number of positive integers n where $1 \le n \le 100$ and n is not divisible by 2, 3 or 5. — 6

b. Determine in how many ways can the letter in the word ARRAMGEMEMT be arranged so that
i) There are exactly two pairs of consecutive identical letters. — 6
ii) At least two pairs of consecutive identical letters.

c. A machine that inserts into envelops goes haywire and inserts letters randomly into envelops. What is the probability that in a group of 100 letters
i) no letter is put into the current envelope ii) exactly 1 letter is put into the envelope
iii) exactly 98 letters are put into the correct envelope
iv) exactly 99 letters are put into the envelope v) all letters are put into the correct envelopes. — 8

7 a. Determine the generating function for the number of n-combinations of apples, bananas, oranges and pears wherein each n-combination the number of apples is even, the number of bananas is odd, the number of oranges is between 0 and 4 and there is at least one pear. — 6

b. In how many ways can a police captain distribute 24 rifles shells to four police officers so that each officer gets at least three shells, but not more than eight? — 6

c. Determine the number of integral solutions of the equation
$x_1+x_2+x_3+x_4=18$
Subject to $1 \le x_1 \le 5, -2 \le x_2 \le 4, \quad 0 \le x_3 \le 5, \quad 3 \le x_4 \le 9$ — 8

8 a. A bank pays 6% annual interest on savings compounding the interest monthly. If you deposit Rs.1000 on the first day of May, how much will this deposit be worth a year latter. — 4

b. Solve the recurrence relation
$F_{n+2} = F_{n+1}+F_n$, where $n \ge 0, F_0 = 0, F_1 = 1$ — 8

c. using only the three letters *a, b, c* words of length *n* are to be transmitted over a communication channel subject to the condition, that no word in which two a's appear consecutively is to be transmitted. Find the number of words allowed by the communication channel — 8

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